

Toward the Unification of Physics and Number Theory

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In Part I, we introduce the notion of *simplex-integers* and show how, in contrast to digital numbers, they are the most powerful numerical symbols that implicitly express the information of an integer and its set theoretic substructure. In Part II, we introduce a geometric analogue to the primality test that when p is prime, it divides $\binom{p}{k} = \frac{p \cdot (p-1) \cdots (p-k+1)}{k \cdot (k-1) \cdots 1}$ for all $0 < k < p$. Our geometric form provokes a novel hypothesis about the distribution of prime-simplexes that, if solved, may lead to a proof of the Riemann hypothesis. Specifically, if a geometric algorithm predicting the number of prime simplexes within any bound n -simplexes or associated A_n lattices is discovered, a deep understanding of the error factor of the *prime number theorem* would be realized – the error factor corresponding to the distribution of the non-trivial zeta zeros. In Part III, we discuss the mysterious link between physics and the Riemann hypothesis (1). We suggest how quantum gravity and particle physicists might benefit from a *simplex-integer* based quasicrystal code formalism. An argument is put forth that the unifying idea between number theory and physics is code theory, where reality is information theoretic and 3-simplex integers form physically realistic aperiodic dynamic patterns from which space, time and particles emerge from the evolution of the code syntax. Finally, an appendix provides an overview of the conceptual framework of *emergence theory*, an approach to unification physics based on the *quasicrystalline spin network*.

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Introductory Overview

The value of this manuscript is in the questions the author hopes are provoked in the mind of the reader:

1. Is there a geometric algorithm that predicts the exact number of prime-simplexes embedded within any n-simplex?
2. If Max Tegmark is correct and the geometry of nature is made of numbers, would they be geometric numbers like *simplex-integers*?
3. Would this explain the correspondence between number theory and physics and support the conjectures of Freeman Dyson (2) and Michel Lapidus (3), who posit the existence of a missing link between number theory and fundamental physics?
4. If nature is made of geometric numbers, how would it compute itself into existence and is the principle of least action an indication it is concerned with efficiency?
5. Are quasicrystal codes maximally efficient?

6. If nature is a symbolic language – a code operating at the Planck scale, can it exist without choosing or “measuring” entities at that scale, as is required to action the syntactically free steps in all codes?

Ontologically, it seems clear that the fundamental elements of reality are made of information. We define *information* here as “meaning in the form of symbolic language”. And we define *language* as “a symbolic code consisting of (a) a finite set of symbols, (b) construction rules and (c) syntactical degrees of freedom. This is in contrast to deterministic algorithms which also use a finite set of symbols and rules but have no degrees of freedom. We define *symbol* here as “an object that represents itself or something else”. And, finally, we define an *object* as “anything which can be thought of”.

Fundamental particles have distinct geometries and are, in some sense, geometric symbols. For example, at each energy state, an orbiting electron forms a finite set of *shape-symbols* – p-orbital geometries composed of the probability distribution of the wave-function in 3-space. There are strict rules on how these fundamental physical geometric objects can relate but there is also freedom within the rules, such that various configurations are allowed. If we speculate that particles are patterns in a Planck scale geometric quantum gravity code in 3D of quantized space and time, we can wonder what the most efficient symbols would be.

Simplexes are efficient symbols for integers and their set theoretic substructure. The prime simplexes within any bound of simplexes, n-simplex to m-simplex, are ordered according to purely geometric reasons. That is, the set theoretic and number theoretic explanation is incidental to the geometric one. Accordingly, speculations by Freeman Dyson (2), Michel Lapidus (3) and others on a hidden connection between fundamental physics and number theory are less enigmatic when considering *shape-numbers*, such as *simplex-integers*. This view brings fundamental physics and number theory squarely into the same regime – geometry – a regime where physics already resides.

As symbolic information, simplexes are virtually non-subjective and maximally efficient when used to express the information of an integer and its set theoretic substructure. A quasicrystalline symbolic code made of simplexes possesses construction rules

and syntactical freedom defined solely by the first principles of projective geometry.

All particles and forces, other than gravity, are gauge symmetry unified according to the E_6 Lie algebra, which corresponds to the E_6 lattice, which can be constructed entirely with 3-simplexes. That is, it can be understood as a packing of 6-simplexes, which can each be constructed from 3-simplexes. More specifically, all particles and forces other than gravity are unified according to the standard model $SU(3) \times SU(2) \times U(1)$ Lie algebra. The single gauge groups that contain this algebra include $SU(5)$ in the form of Georgi-Glashow Grand Unified Theory (GUT), $SO(10)$ (4) and E_6 (5). All three are related by the complex octonion projective plane $(\mathbb{C}x\mathbb{O})P^2$ which is E_6 divided by $SO(10) \times U(1)$ and by the 20-dimensional set of complex structures of 10-dimensional real space \mathbb{R}^{10} , which is $SO(10)$ divided by $SU(5)$. These algebraic objects are isomorphic to their Euclidean geometric analogues, which are simple higher dimensional lattices constructed as packings of simplexes. So E_6 embeds $SO(10)$, which embeds $SU(5)$, which embeds $SU(3) \times SU(2) \times U(1)$. And E_6 embeds in E_8 . In 1985, David Gross, Jeffrey Harvey, Emil Martinec and Ryan Rohm introduced Heterotic string theory using two copies of E_8 to unify gravity with the standard model in an attempt to create a full unification theory. And E_8 can be constructed entirely by arranging, in 8D space, any choice of n-simplexes of equal or lesser dimension than the 8-simplex. Other approaches using E_8 for unification physics include those of G. Lisi (6), T. Smith and R. Aschheim (7).

Cut + projecting a slice of the E_8 lattice to 4D along the irrational hyper vector prescribed by Elser and Sloan generates a quasicrystal. This quasicrystal can be understood as a packing of 3-simplexes in 4D forming super-clusters 600-cells, which intersect in seven ways and *kiss* in one way to form the overall 4D quasicrystal. Because this 3-simplex based object derived from E_8 encodes the E_6 subspace under projective transformation, it also encodes the gauge symmetry unification physics of the *standard model* along with the less well accepted unification of general relativity via E_8 .

A projection encodes the projection angle and original geometry of a pre-projected object. Consider a copy of a unit length line [yellow] rotated by 60° [blue].



Of course, this is a line living in 2D and rotated by 60° relative to a 1D projective space, where it contracts to a length of $\frac{1}{2}$. Similarly, a projection of a cube rotated by some angle relative to a projective plane is a pattern of contractions of the cube's edges, as encoded in the projection, which join to form angles with one another. The total projection is a map encoding the higher dimensional shape plus the rotation of the projector relative to the cube and plane.



One can decode the projection itself to induce the pre-projected cube and the possible projection angles. To form a quasicrystal, more than one cell is projected. A slice of the higher dimensional crystal, called a *cut window*, is projected to the lower dimensional space. The coordinate of the *cut window* can change to generate additional quasicrystalline projections that form animations. These changes can be made by translating and or rotating the *cut window* through the lattice. The coordinates of the various vertex types of the projection will change as the coordinate of the *cut window* changes. The ways these changes can occur are called the *phason* rules and degrees of freedom. This finite set of geometric angles and lengths and the rules and freedom are collectively called the code or language of the quasicrystal.

The aforementioned E_8 crystal can be built entirely of regular 3D tetrahedra – 3-simplexes. When it is projected to 4D, the tetrahedral edges contract but do so equally so that the tetrahedra shrink under projection but remain regular, generating a quasicrystal made entirely of 3-simplexes. As will be discussed later, we then generate a representation of this 4D quasicrystal in 3D.

A quasicrystal is an object with an aperiodic pure point spectrum, where the positions of the sharp diffraction peaks are part of a *vector module* with finite rank. This means the diffraction wave vectors are of the form

$$\mathbf{k} = \sum_{i=1}^n h_i \mathbf{a}_i^*, \text{ (integer } h_i\text{)}.$$

The basis vectors \mathbf{a}_i^* are independent over the rational numbers. In other words, when a linear combination of them with rational coefficients is zero, all coefficients are zero. The minimum number of basis vectors is the *rank* of the vector module. If the *rank* is larger than the spatial dimension, the structure is a quasicrystal (8). And every aperiodic pure point spectrum in any dimension correlates to some quantity of irrational *cut* + *projections* of higher dimensional lattices.

There is an intriguing connection between quasicrystals, prime number theory and fundamental physics. Both the non-trivial zeros of the Riemann zeta function and Eugene Wigner's *universality* signature, found in all complex correlated systems in nature, are pure point spectrums and therefore quasicrystals (9). However, with 1D quasicrystals that possess many nearest neighbor point to point distances, as opposed to the two lengths in the simple Fibonacci chain quasicrystal, it is very difficult to know what “mother” lattices and projection vectors generate them. In other words, we can know it is a quasicrystal because it is an aperiodic pure point spectrum. But we would have no deep understanding of its phason syntax rules or information about the higher dimensional crystals and angles that generated it.

Andrew Odlyzko showed that the Fourier transform of the zeta-function zeros has a sharp discontinuity at every logarithm of a prime or prime-power number and nowhere else (10). That is, the distribution is an aperiodic pure point spectrum – a quasicrystal (2). Disorderly or chaotic non-periodic ordering will not generate an aperiodic pure point spectrum.

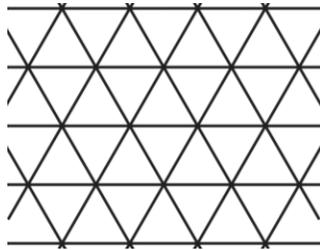
Similarly, for any span of n-simplexes through m-simplexes, the density distribution of simplexes with a prime number of vertices (prime-simplexes) is aperiodic and non-random. Its prime density pattern and scaling algorithm exists for purely geometric reasons. For example, one may consider the 99-simplex. It contains 25 prime-simplexes that have an ordering scheme that drops in density as the series approaches the bound at the 99-simplex. The distribution of the 25 prime-simplexes within this hyper-dimensional Platonic solid is based purely on geometric first principles and is not fundamentally

related to probability theory. Of course, it can be predicted using probability theory. The distribution of prime-simplexes, as *shape-numbers*, within any bound is trivially isomorphic to the distribution of digital integers within the same bound. And the distribution pattern of digital primes is fundamentally non-probabilistic because the identical geometric distribution pattern of prime-simplexes is not probabilistic.

Interestingly, Wigner's ubiquitous *universality* signature describes the quasiperiodic pattern of the zeta zero distribution (11). The pattern occurs in all strongly correlated systems in nature. In fact, it shows up in the energy spectra of single atoms. Indeed, nearly all systems are strongly quantum correlated. Terrence Tao and Van Vu demonstrated universality in a broad class of correlated systems (12). Later, we will discuss more about this pattern, which Van Vu said appears to be yet unexplained a law of nature.

What could possibly correlate the distribution of primes in number theory to something as ubiquitous as the universality signature? As Dyson, recognized, the distribution of prime numbers is a 1D quasicrystal. And the universality signature ubiquitous in physics is too.

Each prime-simplex integer is associated with a crystal lattice as a subspace of the infinite A-lattice. For example, the 2-simplex is associated with the A_2 lattice, which is associated with a crystal made of equilateral triangles and is a subspace of all A_{2+n} lattices.



If we cut + project a slice of this crystal to 1D with the golden ratio based angle of about 52.24° , we generate a quasicrystal code with three “letters” of the lengths 1, ϕ , and $1/\phi$. As we go up to higher dimensional A-lattice crystals associated with a given simplex-integer and project to 1D, the number of lengths or “letters” of

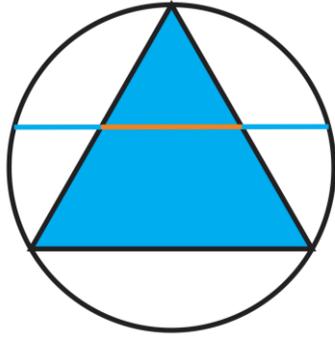
the quasicrystalline code increases. Within any irrational projection of a prime- A_n lattice based crystal, there exists the projections of all A lattice crystals less than n, including a distribution of prime-A-lattices. For example, the projection of the crystal built upon the A_{99} lattice built of the simplex-integer corresponding to the number 100, encodes the distribution of 25 prime- A_n lattice crystals.

This connection can be summarized thusly. (a) The distribution of non-trivial zeta zeros and the distribution of prime numbers is a 1D quasicrystal. (b) all 1D quasicrystals can be derived by irrationally projecting hyper lattice slices. A likely candidate lattice is the one corresponding to simplex-integers. (c) Nature is deeply related to mathematics. (d) The foundation of all mathematics, even set theory, number theory. The appearance of the universality signature in all complex systems and the distribution of primes may relate to the infinite-simplex. That is, the crystal associated with the infinite A-lattice and its projective representation in the lowest dimension capable of encoding information, 1D.

We propose that the missing link between fundamental unification physics and number theory is the study of simplex-integer based lattices transformed under irrational projection, quasicrystalline code theory.

In Part III, we mention that a general feature of non-arbitrarily generated quasicrystals is the golden ratio. Specifically, any irrational projection of a lattice slice will generate a quasicrystal. However, only golden ratio based angles generate quasicrystals with codes possessing the least number of symbols or edge lengths. We will discuss how black hole theory, solid state materials science as well as quantum mechanical experiments indicate there may be a golden ratio based code related to the sought after quantum gravity and particle unification theory. As preparation for that, it is helpful to understand how deeply the golden ratio ties into simplexes.

Of course, the simplest dimension where an angle can exist is 2D. And the simplest object in 2D is the 2-simplex. Dividing the height of a circumscribed 2-simplex with a line creates one long and two short line segments. The ratio of the long to the short segments is the golden ratio.



In fact, this is the simplest object in which the golden ratio exists implicitly, since simply dividing a line by the golden ratio is arbitrary and not implied by the line itself. So there is a fundamental relationship between π and ϕ in the circumscribed 2-simplex.

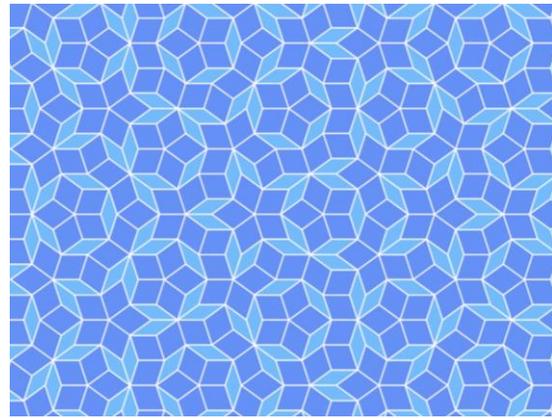
Simplexes are the equidistant relationship between an integer quantity of points and, in their lattice form as packings of tetrahedra, they correspond to periodic maximum sphere packings. For example, the maximum sphere packing in 3D includes the FCC lattice, which is a packing of 3-simplexes. The points where the spheres kiss generate the lattice associated with the 3-simplex called the A_3 lattice. Similarly, the E_8 lattice is a packing of simplexes, and its points are the kissing points of the maximum packing of 8-spheres.

Part I: Proof by Deductive Argument That Simplex-Integers Are the Most Efficient Number Symbols for Integers and Their Set Theoretic Substructure

INTRODUCTION

A *number* is a symbol used to measure or label. Generally, a symbol is an object that represents itself or another object. For example, an equilateral triangle (as the delta symbol) often represents the object called “change” or “difference”.

However, an equilateral triangle (or any object) can also serve as a symbol to represent *itself*, the equilateral triangle. Symbols can be self-referential and participate in self-referential codes or languages. An example is a quasicrystal, such as the Penrose tiling derived by projecting a slice of the 5-dimensional cube lattice, Z_5 , to the plane (13). It is a language (14) because it possesses (a) a finite set of symbols, (b) construction rules and (c) degrees of freedom – called *phason degrees of freedom*.



The Penrose tiling quasicrystal, derived via cut + projection of a slice of the Z_5 lattice, is a code because it contains a finite set of geometric symbols (two rhombs), matching rules and degrees of freedom.

It has two “letters” which are two rhomboid shapes. The rhombus symbols can only be arranged according to specific assembly rules to form seven different vertex geometries that can be thought of as the “words” formed by the two building block geometric “letters”. But within the syntax constrains, there are also degrees of freedom, called *phason* degrees of freedom. The quasicrystalline code is a language in every sense of the word, conveying the meaning of geometric form such as dynamical quasiparticle waves and positions. Yet the geometric symbols, the building blocks of this code, represent themselves, as opposed to ordinary symbols which represent other objects.

It has been shown how 3-simplexes (15) can be the only, shape in non-space-filling quasicrystals in 3D or 4D. All quasicrystals are languages, not just the Penrose tiling. And the 3-simplexes, i.e., simplex-integers, in these special quasicrystalline symbolic codes represent themselves.

ULTRA-LOW SYMBOLIC SUBJECTIVITY

Generally, symbols are highly subjective, where meaning is at the whim of the language users. However, simplexes, as geometric number and set theoretic symbols, which represent themselves, have virtually no subjectivity. That is, their numeric, set theoretic and geometric meaning is implied via first principles. If space, time and particles are pixilated as

geometric code, low subjectivity geometric symbols with code theoretic dynamism such as these could serve as quanta of spacetime.

SIMPLEXES AS INTEGERS

The geometric structure of a simplex encodes numerical and set theoretic meaning in a non-arbitrary and virtually non-subjective manner. For example, the digital symbol “3” does not intrinsically encode information about the quantity of three objects. In fact, any object can serve as a symbol for a number. So we introduce the notion of *simplex-integers* as virtually non-subjective symbols for integers.

Symbolic Function 1: Counting

The number of 0-simplexes in a given n-simplex indexes to an integer. For example, the 2-simplex has three points or 0-simplexes corresponding to the digital symbol “3”. The most basic information of an integer is its counting function. The series of simplex-integers counts by adding 0-simplexes to each previous simplex-integer symbol.

Symbolic Function 2: Set Theoretic Meaning

Inherent to a integer is its set theoretic substructure. A simplex-integer is a number symbol that encodes both the counting function and set theoretic substructure of an integer. For example, the quantity of four objects can be communicated by the symbols 𐤄, 4 or IV. However, when we use a 3-simplex to represent the counting function of the number 4 we also encode its set theoretic substructure:

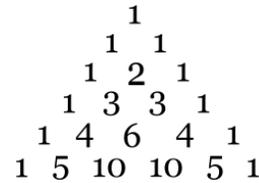
- Four sets of one
- Six sets of two
- Four sets of three
- One set of four

This is geometrically encoded in the 3-simplex as four 0-simplexes (points), six 1-simplexes (edges), four 2-simplexes (faces) and one 3-simplex (tetrahedron).

Symbolic Function 3: Binomial Expansion

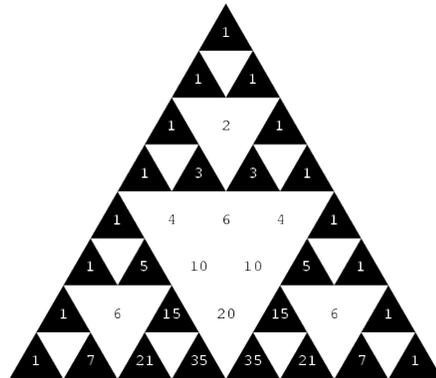
An n-simplex encodes the binomial coefficient corresponding to a row of Pascal’s triangle.

The coefficients are given by the expression $\frac{n!}{k!(n-k)!}$
Pascal’s triangle is the arrangement into rows of successive values of n . The k ranges, from 0 to n , generate the array of numbers. It is a table of all the binomial expansion coefficients.



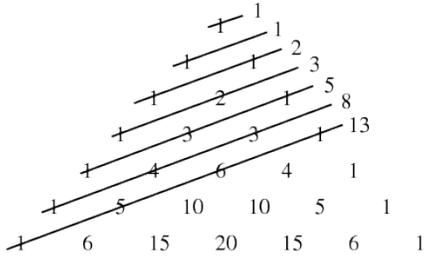
Symbolic Function 4: Sierpinski Triangle Fractal

Because each simplex is a higher dimensional map of the 2D Pascal’s triangle table, it too encodes the same Sierpinski triangle fractal when the positions of the table are coded in a binary fashion to draw out the odd and even number pattern, as shown below with fractal dimension $\log(3)/\log(2)$. This same fractal can be a cellular automaton generated by Rule 90 (16), the simplest non-trivial cellular automaton (17). Specifically, it is generated by random iterations of the time steps of Rule 90.



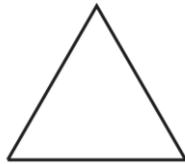
Symbolic Function 5: Golden Ratio in the Infinite-Simplex

Pascal’s triangle is analogous to a matrix representation of the sub-simplex sums within any n-simplex. Diagonal cuts through Pascal’s triangle generate sums that are successive Fibonacci numbers. Any two sequential Fibonacci numbers are a close approximation of the golden ratio. The series of ratios converges to the golden ratio.



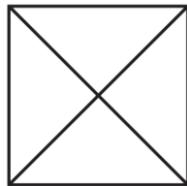
SYMBOLIC POWER OF SIMPLEX INTEGERS

In graph theory, one can use a graph drawing as a numerical symbol to count quantities of objects and explore their set theoretic relationships (connections). This makes a graph diagram analogous to the counting function and set theoretic substructure function of a simplex-integer (18). For example, the complete and undirected graph of three objects expresses the set theoretic substructure implied by the integer 3. The graph drawing symbol is usually the 2-simplex.



A key aspect of this symbol is the equidistance between its points – its connections. The complete and undirected network (graph) of three objects has no magnitudes in its connections. To geometrically represent the complete graph of three objects, equidistance symbolizes the notion of equal magnitude of the graph connections. So the 2-simplex is an efficient or *waste-free* symbol for the complete and undirected graph of three objects.

The diagrammatic symbol graph theorists typically use for the complete and undirected graph of four objects is given below. It encodes the full set theoretic substructure of the integer 4.



But here we see a breakdown in efficiency because we have *wasted* information in the drawing or symbol. It

does not inherently represent the notion of equal connection magnitude because four connections have one length and two have a longer length. This superfluous information in the symbol must be ignored by the graph theorist. It is wasted.

The only way to have equidistance between four points in a geometric symbol is to extrude an additional spatial dimension – to go to 3D. In this case, the tetrahedron can symbolize, in a non-subjective and waste-free manner, the equidistant relationship of four points and their full set theoretic substructure. This is the case for all simplexes, where each encodes a positive integer and its full set theoretic substructure in the most efficient manner possible without wasted symbolism, and where all connections the same length or magnitude.

Of course, we cannot make symbols in spatial dimensions greater than three. However, we can work with higher dimensional simplex symbols in the form of their associated geometric algebras¹.

Next, let us sketch out a proof that simplex-integers are the most powerful numbers to express counting function and set theoretic substructure.

A METHOD FOR RANKING SYMBOLIC POWER

Here, the term *symbolic power* shall be synonymous with *symbolic efficiency*. Our discussion is concerned only with the efficiency ranking of symbols that can represent the meaning of (a) integers and (b) their set theoretic substructure. The rank of meaning or information content of an integer and its set theoretic substructure increases with size.

The challenge, is to logically rank of the magnitude of inherent information of a given symbol. Then we can consider the set of all symbols which might encode the numeric and set theoretic meaning of integers to see if there is one type with maximal efficiency.

¹ Each simplex is associated with a Lie group of the series A_n and its Lie algebra a_n , which corresponds to geometry because a Lie group encodes geometric operations (mirror reflections and rotations). Each simplex is associated to a Clifford algebra (commonly named *geometric algebra*), while each sub-simplex is a basis element of the same dimension.

Symbolic efficiency here is concerned with the ratio of:

1. Irreducible sub-symbols
2. Meaning

Because the numeric and set theoretic meaning is established, we are seeking to understand what symbols are the most minimalistic or elegant – the least complex for this purpose. Accordingly, let us discuss the magnitude of a symbol’s complexity.

If a symbol can be reduced to irreducible sub-symbols, it is a composition of some quantity of simpler symbols. We will use that quantity as the magnitude rank of symbolic complexity. For example, the sentence “The dog ran fast” is itself a symbol. But it can be decomposed into clauses, words and letters. Indeed, the letters themselves can be decomposed into simpler subparts as points and connections or lines.

Again, symbols are objects that represent themselves or another object. And in the universe of all objects, the *empty set* and 0-simplex are equally and minimally simple. There can be no simpler object. These are the only two to possess the quality of being non-decomposable into simpler objects. That is, all other symbols are composites of other objects/symbols.

It is difficult to conceive of building composite symbols and a symbolic language out of *empty sets*. Points (0-simplexes), on the other hand, can be arrayed in spaces to form familiar symbols or can be connected graph theoretically without spaces to form non-geometric symbols.

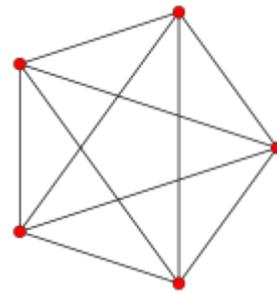
The simplest object in n-dimensions is the n-simplex (19). And every n-simplex is a composite of irreducible 0-simplexes or vertices $\{v_1, v_2, \dots, v_n\}$, where every subset in the structure is a simplex of n-m dimension. Subsets with one element are points, subsets with two elements are line segments, subsets with three are triangles, subsets with four are tetrahedra and so on.

The reason a simplex is the simplest object in any spatial dimension is because it is the least number of non-decomposable symbols (0-simplexes) needed to form a convex hull occupying all sub-dimensions of a given spatial dimension. Of course, the simplex need

not be regular to possess this quality. However, regular simplexes have only one edge length, one edge angle, one dihedral angle, etc. Irregular simplexes possess far more information, where members of a set must be distinct from one another. Accordingly, irregular simplexes possess more information or sub-symbols. For this reason, a regular n-simplex is the simplest object possible in any spatial dimension.

Let us review. It is clear that the irreducible 0-simplex is the simplest object that can form composite symbols. It is clear that compositing a set of them to form a series of symbols called simplex-integers non-arbitrarily and inherently encodes the numeric and set theoretic meaning we are concerned with efficiently and non-arbitrarily symbolizing.

Can we allow, for example, an irregular equilateral triangle to encode the meaning we are interested in and still call it equally simple because it uses the same number of points? It is true that the irregular triangle encodes the numeric and set theoretic meaning. It is also true that a graph diagram in 2D for, say, 5 objects encodes the same information we need with the name number of points.



However, as a symbol, it inherently possesses two different connection lengths. It has additional information not required or needed in our attempt to symbolize the number five and its set theoretic substructure corresponding to equal magnitude of connections. One must ignore that additional inherent meaning of two connection magnitudes implied by the symbol. We may consider this as an equation, where the right side is the meaning of the quantity of five objects and their set theoretic substructure. The left side is a package of inherent geometric information in the symbol we are considering to equal the right side of the equation. We start by counting the quantity of

irreducible 0-simplexes on left side – the symbol side. Counting them gives the value 5. If we allow any irregular 5-simplex, it leaves us with an infinite number of geometric configurations of five points with more than 1 connection magnitude. We wish to minimize the left side further to find the one configuration that is simpler or generates the least amount of unneeded inherent geometric symbolism/information. Specifically, we need all connections between points to be equal. Otherwise, additional information accumulates on the left side of the equation – the inherent information of the symbol itself. Only an n-simplex can achieve this task for any quantity of points. Via this logic, simplex-integers are the most powerful numbers to encode the numeric and set theoretic meaning of the integers.

PART I CONCLUSION:

The simplest object in n-dimension is the n-simplex. The simplest object in any dimension and the only non-decomposable or non-composite symbol is the 0-simplex. The simplest set of composite symbols is the n-simplex series, which adds one 0-simplex to each successive member of the set. The count of 0-simplexes serves as a number – a geometric symbol representing the counting function of an integer and its inherent set theoretic substructure.

Adding more points in some lower dimension, such as 2D, can also serve to as a simple counting symbol that also encodes the set theoretic substructure in the form of the connections on a complete graph drawing. However, this symbol deviates from the pure implied meaning in the simplex-integer series because, without extruding an additional spatial dimension for each added 0-simplex, the connections of the graph drawing take on different length values. The implied information and unnecessary complexity of the symbol breaks down with this more mathematically complex object, where the equal set theoretic “connections” are no longer intrinsically implied with virtual non-subjectivity. One must ignore this extra information and subjectively and arbitrarily interpret the various connection lengths as having equal magnitude.

We prove through this deductive argument that the n-simplex series is the most powerful set of symbols to represent the integers and their set theoretic

substructure. A given simplex embeds the full set theoretic and numeric information of all simplexes within it, including the distribution of prime simplexes isomorphic to the distribution of prime numbers on the ordinary number line. Accordingly, the infinite simplex is the most powerful representation of the integers, prime distribution and the set theoretic substructure of each integer.

Part II: A Geometric Primality Test and the Prime-Simplex Distribution Hypothesis

INTRODUCTION

It is generally believed that if an exact expression were found which determines the number of primes within any bound of numbers, it would lead directly to a proof or disproof of the Riemann hypothesis. This is because the non-trivial zeros that fall on the critical line on the complex plane in the Riemann hypothesis correspond to the *error terms* created by using inexact expressions to estimate the number of primes within a span of integers. Inexact expressions are all that have been discovered so far in prime number theory. An exactly expression is still outstanding and would lead directly to a proof of the Riemann hypothesis if discovered.

There are 25 primes in the bound of numbers 2 to 100. However, the prime number theorem expression of Carl Friedrich Gauss of the form $\pi(x) \sim x / \log x$ (20) incorrectly predicts 21.7 primes in the same bound. And the expression of Peter Gustav Lejeune Dirichlet of the form $\text{Li}(x) \sim \pi(x)$ (21) incorrectly predicts 30.1 primes.

The delta between these inexact approximations and Riemann’s exact function result in error terms that can be written in terms of the non-trivial zeros of the Riemann zeta function. Number theorists have not proven the Riemann hypothesis because they do not deeply understand these non-trivial zeros. That is, they do not understand how to exactly predict the distribution of primes within a bound of integers.

$$\pi_0(x) = R(x) - \sum_{\rho} R(x^{\rho}) - \frac{1}{\ln x} + \frac{1}{\pi} \tan^{-1} \frac{\pi}{\ln x} \quad (1)$$

Because an analytical expression for the second term in equation (1) does not exist, this term quantifies the

error in prime counting functions. It is a sum over the non-trivial zeros of the Riemann zeta function (ρ) on the critical line. The Riemann hypothesis states that all non-trivial zeros lie on the critical line. Riemann's formula is exact if and only if the Riemann hypothesis is true. Again, mathematicians have not proven the Riemann hypothesis because they do not deeply understand the non-trivial zeros. If an exact form for $\pi_0(x)$ is found that does not depend on the zeros, it could be used to shed light on their nature and should lead to a solution of the Riemann hypothesis.

Mathematics is like an *upside down pyramid*, where sophisticated math is built upon a foundation of simple math. And base math is the counting numbers. For example, before one can think about set theory, one must possess a notion of counting numbers. A huge collection of potential proofs exists in the literature. Essentially, they state, "If the Riemann hypothesis is true, we can prove _____." (22). With the current state of prime number theory, we have somehow missed something deep. It is trivially true that within the geometric structure of the infinite-simplex and its irrational projection to 1D, the distribution of prime-simplex and prime digital numbers is encoded. Accordingly, this "deep" aspect of prime number theory, and therefore all mathematics and mathematical physics, which we have missed is geometric number theory.

GEOMETRIC PRIMALITY TEST

We introduce a geometric analogue to the primality test that when p is prime, it divides,

$$\binom{p}{k} = \frac{p \cdot (p-1) \cdots (p-k+1)}{k \cdot (k-1) \cdots 1} \text{ for all } 0 < k < p.$$

Our geometric form provokes the *prime-simplex distribution hypothesis* that, if solved, leads to a proof of the Riemann hypothesis.

CLAIM: *If and only if the quantity of vertices of an n -simplex is evenly divisible into each quantity of its sub-simplexes is that simplex a prime-simplex and associated with a prime A -lattice.*

$$p \text{ is prime} \Leftrightarrow (\forall k \in \mathbb{N}^*, k < p \Rightarrow p \mid \#\{S_k \subset S_{p-1}\})$$

$$p \text{ is not prime} \Leftrightarrow (\exists k \in \mathbb{N}^*, k < p, (\#\{S_k \subset S_{p-1}\})[p] \neq 0)$$

Note: $\#\{S_k \subset S_{p-1}\}$ is the binomial coefficient $\binom{p}{k} = \frac{p!}{k!(p-k)!}$

We want to prove that p is prime if and only if p divides into C , where C is given by equation (X). $C = \frac{p!}{k!(p-k)!}$ for any k between 2 and $p-1$. C satisfies this equation: $p! = C k! (p-k)!$

First, we demonstrate that, if p is prime, p divides C .

Conjecture 1: (if p is prime, p divides C)

Proof : Because p divides $p!$, p also divides one of the three factors on the right side: C or $k!$ or $(p-k)!$

$k < p$ and $k!$ is a product of numbers smaller than p . Therefore, p does not divide $k!$. If k is greater than 1, $(p-k)!$ is a product of numbers smaller than p . Therefore, p does not divide $(p-k)!$. So, necessarily, p divides C . QED.

Conjecture 2 : if p is composite, let $p=a b$, where $a \neq 1$, $b \neq 1$ then at least one of the coefficients is not divisible by p .

Next, we demonstrate that, if p is composite, let $p=a b$, where $a \neq 1$, $b \neq 1$ then at least one of the coefficients is not divisible by p .

$$\text{Take } k = a : C = \frac{(a b)!}{a!(a(b-1))!}$$

We can rewrite as $C = b(a b - 1)(a b - 2) \dots (a b - a + 1)/(a-1)!$

C is not divisible by $a b$, because none of the factors $(a b - 1), (a b - 2) \dots (a b - a + 1)$ is divisible by a , and b is not divisible by $a b$. QED

[Credit goes to Raymond Aschheim for assistance with the above the equations.]

PRIME-SIMPLEX DISTRIBUTION HYPOTHESIS

When studied as simplex-integers instead of digital integers, there is a simple formula that separates prime numbers from composite numbers. That is,

there is a non-constant polynomial that takes in only prime values.

There is no known formula that separates primes from composite numbers. Interestingly however, there exists a purely geometric reason why a given simplex is prime or why there are, for example, 25 prime simplexes embedded in the 99-simplex. The reason is not directly number or set theoretic. Number and set theoretic aspects are merely incidental or secondary to the geometric structure. The reason is based solely on the first principles of Euclidean geometry.

An extension can be made to A-lattices, which correspond to simplexes. For example, the 4-simplex corresponds to the A_4 lattice, which embeds the A_2 and A_3 lattices. Using the 99-simplex example again, the A_{99} lattice, built as a packing of 99-simplexes, embeds 25 prime- A_n lattices and describes their distribution exactly without probability theory based approximations.

The unknown formula expressing the drop in prime-simplex and prime-A lattice density within some bound is also purely geometric. In our future work, we intend to focus on this problem. However, we can state with certainty that the algorithm can be expressed with quasicrystalline formalisms when studied via the irrational projective transformation of a slice of the infinite-A-lattice.

The vertices of a prime-simplex are evenly divisible (without a remainder) into each sum its sub-simplexes. When one considers what this means in terms of shape analyses, such as symmetry or topology, it becomes clear that there must be special shape qualities present in prime-simplexes that are not evident in non-prime-simplexes. For example, the 3-simplex is the first to fail this geometric primality test. Its sub-simplex quantities are:

4 *0-simplexes*
 6 *1-simplexes*
 4 *2-simplexes*

Its four vertices do not evenly divide into its six edges. By contrast, when we look at the simplex-integer associated with the prime number 5, we see sub-simplex sets of:

5 *0-simplexes*
 10 *1-simplexes*
 10 *2-simplexes*
 5 *3-simplexes*

For lack of a better term, there is a *division symmetry* in this simplex with respect to its geometric parts. The “beauty” of 5 vertices evenly dividing into the sums of each sub-simplex inspires the curiosity about what special volumetric, topological or symmetry qualities this shape possesses.

PRIME NUMBER DISTRIBUTION AND FUNDAMENTAL PHYSICS

As far as impacting science, the discovery of the actual algorithm predicting the distribution of primes simplexes within an n-simplex may have important implications for fundamental physics, shedding light on an equally monumental outstanding problem: the *theory of everything* that unifies the theory of space and time (general relativity) with the theory of the quantum world (quantum mechanics).

It is certainly true that nature is deeply mathematical, which means its foundation is built upon counting numbers. But nature is deeply geometric as well. So geometric counting numbers, like simplex-integers, are interesting.

As discussed in Part II, there is a mysterious connection between physics and the distribution of prime numbers.

Inspired by the Hilbert-Polya proposal to prove the Riemann Hypothesis we have studied (23) the Schroedinger QM equation involving a highly non-trivial potential whose self-adjoint Hamiltonian operator energy spectrum approaches the imaginary parts of the zeta zeroes only in the critical line.

$$S_n = \frac{1}{2} + i\lambda_n$$

This is consistent with the validity of the Bohr-Sommerfeld semi-classical quantization condition. We showed how one may modify the parameters which define the potential, fine tuning its values, such that the energy spectrum of the (modified) Hamiltonian matches all zeroes. This highly non-trivial functional form of the potential is

found via the Bohr-Sommerfeld quantization formula using the full-fledged Riemann-von Mangoldt counting formula for the number $N(E)$ of zeroes in the critical strip with imaginary part greater than 0 and less than or equal to E .

Our result shows a deep connection between the most foundational model we have for reality, quantum mechanics, and prime number theory.

Patterns in nature over time or space can only be of three fundamental species:

1. Periodically ordered
2. Aperiodically ordered
3. Random

There is no solid evidence for randomness in nature. In fact, demonstrating it is impossible because one cannot write it down, as can be done with periodic and aperiodic patterns. An experimentalist can only concede she has not been able to find periodic or aperiodic order. The lack of finding order is not good experimental evidence for the theory of randomness. What does have good supporting experimental evidence is the theory of non-determinism, which fits our *code theoretic axiom* (24). For example, in 1984, Dan Shechtman reported his observation of code-theoretic aperiodic order in a material *known* by the scientific community to be disorderly – randomly structured (amorphous) (25). The consensus belief was built upon a bedrock of crystallographic mathematical axioms and decades of failure to observe order in this type of material. And yet Shechtman observed the signature of aperiodic order in the material. A good portion of the scientific community, led by Nobel laureate Linus Pauling, rejected his findings due, in part to the popular theory that randomness is real (26).

Similarly, number theorists have no idea why or how the quasiperiodic spectrum of the zeta zeros possesses the *universality* spectral pattern. Some mathematicians think there may be an unknown matrix underlying the Riemann zeta function that generates the universal pattern. Paul Bourgade, a mathematician at Harvard, said, “Discovering such a matrix would have big implications for finally understanding the distribution of the primes” (27).

So why would proving the Riemann hypothesis help in the search for a *theory of everything*? Because there is a unifying principle in the form of (1) simplex-integers and (2) quasicrystal codes based on simplexes. The notion of randomness in physics would become an old paradigm giving way to the new ideas of aperiodic geometric- language based physics and the *principle of efficient language* (24). Non-deterministic syntactical choice would replace randomness as the ontological explanation for non-determinism. Discreteness would replace the older notion smooth space and time. Number and geometry would be unified via the mathematical philosophy of *shape-numbers*, where nature is numerical – simplex-integers forming the substance of reality – geometry, all within a logically consistent self-actualized code-theoretic universe.

ARE DIGITAL NUMBERS A DEAD-END APPROACH TO PRIME NUMBER THEORY?

Clearly, the *universality* aspect of complex physical systems is deeply rooted in the geometry of particles and forces acting in 3-space. And prime and zeta zero distribution displays the same quasicrystalline pattern.

Non-geometric methods, such as probability theory and brute force computational methods, are typical tools for modern number theorists working on prime number problems. If the 2,300 years of stubbornness of this prime distribution problem has been because it is a deeply geometric challenge, then we have been using the wrong tools for a long time.

As mentioned, within, for example, the 99-simplex, there are 25 prime-simplexes, which are simplexes with a prime quantity of vertices. The reasons for why this bound of simplexes 2 to 99 has a density of prime simplexes of 25 is a purely geometric problem, even though the solution is exactly the same as the unknown algorithm determining the exact quantity of prime digital numbers in the same bound. In other words, the algorithm determining the distribution of prime-simplexes in some bound is the missing and correct algorithm that encodes the *error term*, i.e., it equals the *error term* plus the incorrect result of solutions using the prime number theory algorithm or others.

A fresh and little-focused-on approach is to move prime number theory problems from digital number theory into the domain of simplex-integer number theory – into the realm of pure hyper-dimensional Euclidean geometry and its associated geometric algebras and moduli spaces.

The purely geometric algorithm that determines the number of prime-simplexes within an n-simplex is knowable. It would give the exact number of prime numbers within any span of integers. But what is so deeply different about the digital versus simplex-integer approaches? Two core things: (1) non-transcendental convergence and (2) non-homogeneity of sequential number deltas.

NON-TRANSCENDENTAL CONVERGENCE

Digital integers do not converge when you add them as a series. And divergences are unhelpful because they tell you little – that is, they don't give you a number because they explode toward infinity. In order to convert digital integers into a convergent series, one applies the zeta function. For example, we put a power on each integer and then invert it. We do this with the next integer and add to the previous solution. We repeat this with all integers to transform the integers into a convergent additive series that tells us something deep about the integers and their fundamental skeleton, the primes:

$$1/(1^2) + 1/(2^2) + 1/(3^2) \dots = \pi^2/6$$

What is remarkable is that π is a deeply geometric number even though integers do not appear to be associated with geometry. It is generally believed that π is transcendental, although this is debated (28). The two most famous transcendental numbers are π and the basis of the natural logarithm, e .

Both e and π are deeply geometric. The exponentiation is the fundamental operation to transform an angle θ into a complex number $\text{Exp}(i \theta)$ which, multiplied by a vector, also expressed as a complex number, operates the rotation of this vector from this angle. The constant e is defined by the choice of radian as a unit for the angle, which sets π to measure a half circle rotation by $\text{Exp}(i \theta) = e^{i \theta}$, or $e = \text{Exp}(1)$. This also involves $I = \sqrt{-1}$.

Both e and π fundamentally relate to the Riemann hypothesis but only when explored via digital numbers.

Specifically, e is part of the false error generating algorithms that imperfectly predict the number of primes in a bound – thereby generating the *error term* that maps to the zeta zeros and perhaps preventing a proof of the Riemann hypothesis. And, π is related by the convergence of the zeta function itself to $\pi^2/6$. The zeta function process is how we plot the zero solutions related to the errors onto the complex number plane.

It may be helpful to inquire, “If the *error term* generating method using digital numbers relates to the transcendental numbers π and e , is the inverse true, where in some sense we can say the use of π and e generate the *error term*?” Although this question is confusing, it cuts deep. In other words, there is little choice when using digital integers – the zeta function using digital integers converges² to π . And e is deeply related to π by similar reasoning associated with the choice to use digital integers – as opposed to simplex-integers.

It is reasonable to realize, though, that π and e are deep aspects of the *error term*. And the *error term* is the problem. It is simply the delta between the imprecise temporary “placeholder” prime density prediction algorithm and the currently unknown imprecise ones.

So is it as simple as that? Can we avoid the error term by avoiding digital numbers and, by extension, π and e ?

We will not see π and e when we attack the problem via simplex-integers. The true algorithm determining the number of primes within a bound is geometric and related to an algebraic number. Specifically, $\sqrt{2}$. Interestingly, we do see a relationship to 2 in current prime number theory based on digital integers. The non-trivial zeros are all on a coordinate at $1/2$ the length of the strip bounded by $-1/2$ on the left and $1/2$ on the right.

² More technically, zeta(x) converges to 0 when x goes to infinite, but zeta(2n) for any positive even integer is expressed as a rational fraction multiplied by π at the power 2n.

As mentioned, the zeta function is a method to uncover something deep about the primes by expressing them as a convergent additive series. The additive series of the simplex-integers is naturally convergent without need to invert values? Consider the circumradius of each n-simplex and index each successive one to a circumradius generated by an n-simplex with a number of vertices equal to that prime number. The circumradius of the 1-simplex is $\frac{1}{2}$. And the circumradius of the infinite-simplex is $1/\sqrt{2}$. The circumradii of all simplexes can be related as a series of concentric circles, each two with a different distance between them than the distance between any other two. The distance between each two corresponds uniquely to a certain prime or non-prime integer such that we may call each delta between sequential circumradii a unique integer. And the sum of all deltas is $1/\sqrt{2}$. Within any span of such rings, there is a subset that are prime based, wherein the pattern of radii are neither periodically nor randomly arrayed. They are arrayed as a quasicrystal.

Here we see the first example of a radically different form of convergence in simplex-integers, where the convergence value is an algebraic number instead of a transcendental number like $\pi^2/6$.

NON-HOMOGENEITY OF SEQUENTIAL NUMBER DELTAS

A key difference between digital and simplex-integers is the information encoded in the delta between successive numbers. For example, a few of the geometric deltas between two successive simplexes are:

- Dihedral angles (series ranges from $\text{ArcCos}(\frac{1}{2})$ to $\text{ArcCos}(0)$).
- Circumradii
- Hyper-volumes

Again, the deltas index to integers. The delta between the circumradius of the 1-simplex and 2-simplex would index to the integer 3 because the 2-simplex corresponds to 3 vertices or 0-simplexes.

The salient point is that the delta between two successive simplex-integer geometric indexes is unique

and different than the delta between any other successive pair of simplex-integers.

By contrast, the difference between any two successive digital integers is always 1. Accordingly, it is homogeneous and therefore gives absolutely no number theoretic information. This rich extra information of simplex-integers provides a wealth of geometric clues to fuel a new approach to search for the exact scaling algorithm for density distribution of *geometric primes* as prime-simplexes – the deep reason for why a prime-simplex shows up every so often in a given series of simplexes. And this answer is trivially isomorphic to the distribution of prime digital integers.

EXTENSION TO LATTICES AND GEOMETRIC ALGEBRAS

Consider a 2-simplex, the equilateral triangle. Around it, there can be an infinite 2-simplex lattice, called the A_2 lattice (29). This is a tiling of the plane with equilateral triangles. The lattice associated with the simplex-integer 4 is the lattice described by a maximum density packing of spheres in 3D – the way oranges are stacked in the supermarket. This is called the A_3 lattice and is composed by rotating A_2 lattices from one another by $\text{ArcCos}(1/n)$, where n is the integer corresponding to the A_2 lattice (in this case, 3). This continues ad infinitum, where, for example, the lattice associated with the integer 100, the 99-simplex lattice called A_{99} , is a stack of irrationally rotated parallel lattices A_2 through A_{98} .

We can extend the idea of prime-simplex distribution to prime-simplex-A-lattice distribution. Each n-simplex can pack to form a crystal of n-simplexes – compositing to the A_n -lattice for a given dimension. We can then algebraically explore the reasons for why prime A-lattices appear where they do in a given span or *stack*.

A given A-lattice is associated with various geometric algebras, such as Lie and Clifford algebras. And the geometric algebra of a given A-lattice contains an *algebraic stack* of sub-algebras associated with each sub-A-lattice. This algebraic space corresponds to a point array and is called a *moduli space* (30). These geometric algebra tools can be used to work on the geometric problem of finding the actual and precise

scaling algorithm for the density distribution of prime-simplex associated A-lattice geometric algebras within a larger stack of algebras – again, an algorithm identical to the unknown precise density algorithm for the distribution of prime digital numbers, which would immediately lead to the proof (or disproof) of the Riemann hypothesis.

The “writing on the wall” seems clear. The 2,300 years of searching for the correct prime distribution algorithm via digital numbers and the 157 years of mathematicians trying to prove the Riemann hypothesis via digital numbers are impressive. This apparent roadblock supports the argument that the solution can only be found within the realm of geometry. In addition, the geometric physical connections and the geometry of π and e make it even more reasonable to surmise that digital number theory and stochastic approaches will not lead to an answer.

Because only prime simplex vertices divide evenly into all quantities of sub-simplex, there is an aspect of these special prime shapes that is different than non-prime simplexes. This geometric difference has additional aspects other than the pure set theoretic qualities of the divisibility of vertices. It is analogous to the fact that an equilateral triangle contains all the set theoretic information of the complete and undirected graph of three objects. However, it also contains additional shape related geometric information that goes beyond the set theoretic information of the graph. Prime-simplex-integers possess special group theoretic qualities, topological qualities and various geometric qualities that set them apart from non-prime-simplexes.

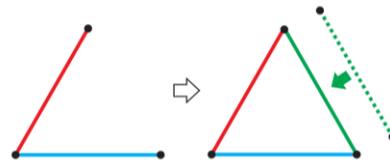
GRAPH-DRAWINGS

As discussed, one may gain intuition by understanding simplexes and A-lattices via a growth algorithm that generates a graph *drawing*, where the quality of the connection magnitudes are lengths, making the object a graph drawing because the connection types are geometric – line segments. Abstractly, this can be true without admitting a smooth spatial substrate, such as \mathbb{R}^3 . In other words, the space is discretized such that there exists no information or space between the line segments of the network. This minimalistic graph-

drawing space is in some sense a quantized subspace of a continuous space.

Picture a line segment and rotate a copy on one end by 60° into another finite 1D spatial dimension to generate the three points of a 2-simplex and its associated A-lattice. We can play with semantics by saying we have rotated a copy of the finite length 1D universe into another finite 1D universe. The objective of the visualization is to disassociate ourselves from the idea of a smooth Euclidean space in a network of finite 1-simplexes, where length is real but restricted to the connections of the network – a graph-drawing made of 1-simplexes.

Or we can index the length value to an abstract graph theoretic magnitude. In any case, the connection relationship between the three points (from the two lines sharing a point) is isomorphic to a 2-simplex.



Next, we rotate a copy of the second line (note that we do not need the green line in the diagram in order to generate the 3 points of the 3-simplex) into the third spatial dimension by 60° from the previous line to generate the 4 points of the 3-simplex and so on. Again, we reject the assumption of a smooth 3-space in favor of an approach that is graph-theoretic.

Each of these 60° rotations from the previous edge is equal to rotating the edge by $\text{ArcCos}[(n - 1)/(2n)]$ from the total simplex construct below it, where n is the simplex number.

As mentioned, this iterative process results in the stack of simplexes converging to a circumsphere with a diameter of $\sqrt{2}$ at the infinite-simplex. When the construction of any simplex, such as the 99-simplex is visualized with 60° rotations extruding successive spatial dimensions, one realizes that the lines form a hyper-dimensional discretized *non-Archimedean* spiral.

In other words, the circumradii of the simplex series starts at $\frac{1}{2}$ for the 1-simplex and converges at $1/\sqrt{2}$ at the infinite-simplex. This is in stark contrast to the cubic series of circumradii, which is divergent, converging at an infinite radius for the infinite-cube. So from the simplex-integer 2 (1-simplex) to the infinite-simplex, the increasing radii values must be distributed over a distance of $1/\sqrt{2} - \frac{1}{2} \approx 0.207$. But in just the distance from the circumradius of the 1-simplex to the 2-simplex, we cover about .077 or more than 37% of the total 0.207 distance to convergence – a distance that must be distributed over an infinite number of simplexes.

Of course, this rapid convergence is massively exponential.

The distribution of prime numbers on the digital number line drops with distance. For years, number theorists have used the prime number theory algorithm (and improved versions) related to the natural logarithm number e to predict the number of primes within a given bound. This correlates to a non-Archimedean spiral called the logarithmic spiral because the distances between turnings increase in geometric progression as opposed to an Archimedean spiral.

Is it the natural logarithm number $e \approx 2.718$ that corresponds to the *actual* algorithm for prime distribution – the one that does not generate the *error term*? As mentioned, e is an artifact of the exploration of the problem via digital numbers and is fundamentally part of the *error term*. That is, the algorithm for predicting prime density that is related to e simply does not work. It is the chosen formalism of the approximation itself that generates the *error term* corresponding to the non-trivial zeros. The logarithmic spiral correlated to the distribution of prime-simplexes should logically relate to a hyper-spiral corresponding algebraic irrational $\sqrt{2}$. So, just as the prime number theorem algorithm corresponds to the non-algebraic transcendental number e , the simplex-integer prime number algorithm for prime distribution would correspond to the algebraic number $\sqrt{2}$ (and to the golden ratio by arguments beyond the scope of this paper).

CREATION OF A PRIME NUMBER QUASICRYSTAL

There is an interesting approach we will explore in a subsequent publication. We will cut+project various A-lattices to lower dimensional quasicrystals and do spectral analyses on the Fourier transform of each. We predict prime-A-lattice associated quasicrystals will possess distinct spectral signatures. If so, a spectral analysis of the superposition of a span of simplex-integer associated A-lattices projected to lower dimensional quasicrystals is expected to reveal the signature of a prime A-lattice distribution scaling algorithm. Such an algorithm would predict the exact prime-simplex density distribution for any bound of projected A-lattices.

PART III CONCLUSION:

We have established a hypothesis, which should be true by trivial deduction. An exact algorithm for the distribution of primes exists in the realm of pure geometry.

Part III: Simplex-Integer Unification Physics

If nature were a self-organized simulation, it would be a simplex-integer based quasicrystalline code derived from E_8 .

INTRODUCTION

The *digital physics* (31) view is the idea that reality is numerical at its core [See work by Ed Fredkin (32), Toffoli (33), Wolfram (34), and Wheeler (35)]. But the numbers need not be digital. They can be *shape-numbers*, such as *simplex-integers*. And because reality is geometric and has three spatial dimensions, one could surmise the following: If nature were built of 3D *bits* of information that are also numbers, the most powerful candidate for a 3D geometric number is the 3-simplex. *Power* in this context is synonymous with *efficiency* in the manner explained in Part I.

Higher dimensional lattices, such as E_6 and E_8 that are associated with unification physics, can be constructed entirely from 3-simplexes. Certain projective transformations result in the 3-simplexes remaining regular but being ordered into a quasicrystalline code

that encodes the higher dimensional lattices and associated gauge symmetry physics.

A hallmark and general characteristic of quasicrystals is the golden ratio. For example, the simplest quasicrystal possible is the two length Fibonacci chain, as 1 and $1/\Phi$. Virtually all 3D quasicrystals found in nature are golden ratio based with icosahedral symmetry.

A self-organizing code on an abstract quasicrystalline substrate is in some sense like a computer but better described as a neural network. Computer theory is concerned with the efficiency of *creating* information in the form of solutions to problems. Information theory is concerned with the efficiency of information *transfer*. Neural network theory (36) is concerned with both the efficient *creation* and *transfer* of information in a network. Neural networks operate via codes, i.e., non-deterministic algorithmic processes – languages. Neural networks in nature are spatial (geometric) arrays of nodes with connections, such as particles connected non-locally by quantum entanglement or forces. And clearly nature, like a neural network, accomplishes the dual task of (a) creating new information (computation) and (b) transferring information. So the universe as a whole is a neural network in the strictest sense of the term.

A special quality of neural networks that sets them apart from computers is that they are non-deterministic. If one subscribes to the theory of randomness and does not require a theory to explain the generator of randomness, one can decide that the free choices in a neural network code are random. On the other hand, there is a special cases in physics where human freewill emerges in a biological neural network, which itself emerges from fundamental particle physics and presumably some unknown quantum gravity theory. In this case, the freewill can act on the syntactical choices in the code-theoretic neural network, providing an explanation for the syntactical choices that might be more explanatory than stopping at randomness as the unprovable axiom. It is generally known that physical reality is: (a) non-deterministic and (b) that it creates the emergence of non-random *freewill*, at least in the case of humans. As this is not a philosophical paper, we will simply say that whatever *freewill* is, it is non-deterministic. It behaves similar to the concept of

randomness insofar as being non-deterministic. The difference is that *choices*, as the *actions* of *freewill*, are made with a blend of subjective meaning, perceptions or opinions combined with logic and choices of strategy. So symbolic language and meaning are deep principles embedded in the theory of freewill. Conway and Kochen proved that if freewill is real, fundamental particles have some form of non-random freewill (37).

Another fundamental feature of nature is that, as a network, it is concerned with *efficiency* in the form of the *principle of least action* and similar laws (38). In fact, efficiency may be the most fundamental behavior of reality – leading directly to *Noether's second theorem* about conservation and symmetries in nature (39), conservation laws and from there to the modern gauge symmetries unification physics, such as seen in the standard model of particle physics.

E_8 IN NATURE

The most foundational symmetry of nature unifies all fundamental particles and forces. It can be described thusly:

All fundamental particles and forces, including gravity, are uniquely unified according to the gauge symmetry transformations encoded by the relationships between vertices of the root vector polytope of the E_8 lattice – the Gosset polytope (6).

We have also shown cosmological correlations to E_8 in *Heterotic Supergravity with Internal Almost-Kahler Configurations and Gauge $SO(32)$, or $E_8 \times E_8$, Instantons* (40). However, our general approach is to exploit projective geometry as the symmetry breaking mechanism in a quantum gravity plus particle physics approach, which recovers particle gauge symmetry unification.

The simplest polytope in eight dimensions is the 8-simplex. The E_8 lattice is the union of three 8-simplex based lattices called A_8 . This lattice corresponds to the largest exceptional Lie algebra, E_8 . That is, the simplest 8D building block of the Gosset polytope and E_8 lattice is the 8-simplex – the most powerful number that inherently encodes the counting function

of $3^2 = 9$ and its full set theoretic substructure. Nine, incidentally, is the first odd number that is not prime.

E_8 DERIVED QUASICRYSTAL CODE

In *Starobinsky Inflation and Dark Energy and Dark Matter Effects from Quasicrystal Like Spacetime Structures* (41) and *Anamorphic Quasiperiodic Universes in Modified and Einstein Gravity with Loop Quantum Gravity Corrections* (42), we show how quasicrystalline codes can relate to quantum gravity frameworks.

A 4D quasicrystal can be created by projecting a slice of the E_8 crystal (43). This 4D quasicrystal is made entirely of 600-cells, which are each made of 600 regular tetrahedra. So the fundamental 3D *letters* or symbols of this quasicrystal are 3-simplexes. The allowable ways these geometric number symbols can spatially relate to one another is governed by cut+projection based geometry (44), which generates the *syntax* of this non-arbitrary quasicrystalline code. The term *code* or *language* applies because the syntax allows various legal. A language or code must have degrees of freedom within the rules and a finite set of symbols that must be arranged by a *code user* in order to create meaning. Geometric codes, such as quasicrystals, generate geometric meaning, such as waveform and quasiparticle position. The *code user* may emerge from the evolutionary complexity of the system itself and can be as sophisticated as a human consciousness and beyond. Or it can be simple, like the guiding tendency of a tornado to preserve and grow its dynamical pattern for as long as possible in a new physical ontology based on code theory instead of randomness.

Furthermore, how two or more syntactically legal quasicrystals can be ordered in a dynamic pattern or animation has a separate syntax scheme based on how the cut-window moves through the hyper-lattice. That is, all behaviors and rules are part of a code based solely on geometric first principles with no arbitrary ad hoc mathematical contrivances.

The caveat is that a freewill chooser of some form must execute the free choices in the code. This is the case with all codes, whether that be a computer language or

song-language of birds. A general quality of codes is that meaning is not maximized and breaks down when strategic choices in the syntax are replaced by, for example, a pseudo random number generator.

Discovering a fundamental unification model of all particles and forces based on such a geometric first principles code is a worthy but formidable challenge. It would be a *microscopic first principles theory of everything*. Currently, there exists no first principles explanation for the fine structure constant, the speed of light, Planck's constant or the gravitational constant. In other words, there is no known *first principles* unification theory. In fact, the fundamental constants h and G and c are only known to about 4 places after the decimal. The CODATA values, which go out to a few more places after 4, are an averaging of six established experimental methods that all disagree at the 5th place after the decimal.

Syntactically legal configurations of these quasicrystal based *simplex-letters* form simplex-based *words* and *sentences*. In other words, groups of simplexes have emergent geometric meaning – shape and dynamism. And sets of these inherently non-local quasicrystalline *simplex sentence* frames can be ordered into *animated* sequences and interpreted as the physical dynamic geometric patterns of space that have both wave and particle like qualities, a well understood dualistic quality of phason quasiparticles in quasicrystal codes (45).

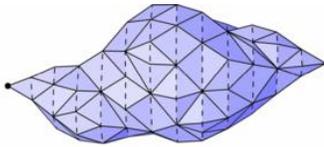
While this part of the discussion is a mix of fact and conjecture, the reader may agree that, if nature is a code or simulation based on *maximally efficient* symbolism, the following three ideas may be at play:

1. Nature must be an efficient symbolic code capable of simulating a 3D reality. Accordingly, the most powerful symbol in 3D – the 3-simplex – may be exploited because it is the simplest and most efficient 3D quantum of information.
2. Spacetime would be discrete, not just, spin, Planck's constant (quantum of action) and the photon (quantum of energy). Nature must have an efficient geometric “pixel” or foundational symbol, just as a dynamic image on a high resolution video monitor is composed of invisible

microscopic building-block pixels or just as a binary computer code is made of irreducible elements symbols, where bytes can be further decomposed into bits but no further. In our proposed framework, the simplex-integer is the irreducible and non-transformable “pixel” in the *simulation* that composes our 3D reality. It is the fundamental shape-symbol in a geometric code/language. In short, simplex-integers “switch-hit” as both numbers and spatial building blocks.

3. It must be a symbolic code derived from E_8 , which encodes the gauge symmetry unification of all fundamental particles and forces. We have generated a 3D quasicrystal language of 3-simplexes derived from E_8 (15).

The *causal dynamical triangulation* program of Amjorn and Loll (46) is encouraging evidence that fundamental physics can be modeled with aperiodic configurations of 3-simplexes as the only building-block element. They have generated very close approximations of Einstein’s *field equations* (47).



QUASICRYSTALS AS MAXIMALLY EFFICIENT CODES

Just as simplex-integers are the most powerful numbers to express counting function and set theoretic information, quasicrystals are the most efficient codes possible in the universe of all codes.

This is a major claim. To understand it, we should first establish the fact that an n-dimensional quasicrystal is a network of quasicrystals in all dimensions lower than it. For example, the Penrose tiling, a 2D quasicrystal, is a network 1D quasicrystals. A 3D Penrose tiling (Ammann tiling (48)) is a network of 2D quasicrystals, which are each networks of 1D quasicrystals.

So the building block of all quasicrystals are 1D quasicrystals. Reducing further, we should understand that there are an infinite set of 1D quasicrystals. The

“letters” of a 1D quasicrystal are lengths. And a 1D quasicrystal can have any finite number of *letters*. However, the minimum is two. The Fibonacci chain is the quintessential 1D quasicrystal. It possesses two lengths related as the *golden ratio*. In order for a quasicrystal greater than 1D to have only two letters, the letters must be 1 and the inverse of the golden ratio. Interestingly, this simple object has a deep relation to E_8 . When a slice of E_8 is projected to 4D according to a non-arbitrary golden ratio based irrational angle (43), the resulting quasicrystal is made entirely of 3-simplexes and is the only way to project that lattice to 4D and retain H_4 symmetry. The angle between adjacent 3-simplexes is $60^\circ + \text{ArcCos}([3\varphi - 1]/4)^\circ = \text{ArcCos}(1/4)$, where φ is the *golden ratio*. This quasicrystal, fully encoding gauge symmetry unification physics, can be described as a network of Fibonacci chains. These are the most powerful 1D quasicrystals for two reasons. As mentioned, the power of a code relates to how many building block symbols it has. This definition of power relates to the discussion earlier, where we spoke of the left side of the equation as being the magnitude of simplicity of the symbolic system. But a code cannot have fewer than two fundamental symbols for obvious reasons. This is what makes binary codes so powerful. Secondly, Fibonacci chain quasicrystal codes are based on the Dirichlet integers 1 and $1/\varphi$, which possess remarkable efficiency characteristics, such as error detection and correction abilities and multiplicative and additive efficiencies. For example, they are closed under multiplication and division. As with all quasicrystals, Fibonacci chains are fractal (49).

The function of division stands out in the arsenal of powers that the golden ratio possesses because it relates to measurement, which is the deepest and most enigmatic aspect of quantum mechanics that is not yet fully understood.

Physics Nobel laureate, Frank Wilczek of MIT said (50), “The relevant literature [on the meaning of quantum theory] is famously contentious and obscure. I believe it will remain so until someone constructs, within the formalism of quantum mechanics, an *observer*, that is, a model entity whose states correspond to a recognizable caricature of conscious awareness.”

Wilczek is speaking of the need to build measurement into a new quantum mechanics mathematical formalism that incorporates an operator capable of measuring at the Planck scale. His motivation is on solid ground. Quantum mechanics indicates that the ontology of physical reality must be based on measurement, where all that exists is that which is measured. Certainly, it is extreme to postulate that physical reality needs humans to measure it in order to be real. A physical formalism based on the premise that reality is made of information in the form of a code would require a quantum scale mathematical operator capable of actualizing information via measurement.

A measurement of any form is ultimately a spatial relationship between the measurer and two additional points in space. This is the case with any detector, such as a human eye or a Geiger counter. Waveforms are reducible to quantum particles. And all detectors are reducible to component particles that interact with signal particles, such as photons, that are emitted from another particle being measured at a distance. For example, a camera takes a photo of a tree by receiving rays of photons that trace to the camera lens. The irreducible measurement, however, is the relationship between a detecting particle and two other particles at two other coordinates. This forms a triangle, where the detector is one vertex and the two measured coordinates are the other vertices. The fundamental information being registered by the detector is a transformation via contraction of the edge length of the triangle that is not connected to the detection particle. For example, you observe two friends in the distance. We can conceptualize you and the two friends as three points in space. There is an actual distance length between your two friends, which we will call L . Because it is impossible for you to have a perfectly equal distance between you and each of your friends, you observe or measure the information of $L-l$, where l is some contraction value on L .

Your transformation of L gives you information about the relationship of your two friends and their relationships to you. If they are standing one car length apart but your angle relative to them is such that you perceive it as $\frac{1}{2}$ a car length, then you intuitively know how to decode that information to tell you their actual distance as one car length plus your position relative to them. Similarly, when you look at the complexity of,

say, a tree, the massive package of information from that measurement/observation is merely a composite of these individual length transformations between pairs of points and the measuring detector, forming a transformation of the pre-transformed triangle.

So L and l form a relationship in your mind as a ratio. The meaningful information of your measurement is not l it is the ratio of L to l , which tells you information about the relationship of the two measured points to one another (their actual length relationship) and their length relationship to you from your vantage point.

Consider this set of three points that are equally spaced in a line in 3-space. If you measure them with your eye from a golden ratio based vantage point equal to a rotation of the line of three points by a golden ratio angle, then you can divide the total length into two parts as 1 and $1/\phi$.

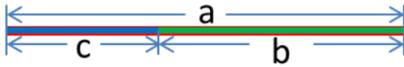


To review, all measurements are divisions or ratios. And a choice of measurement (observation) is necessary to actualize or make-real any information. If we consider that reality is information theoretic or code-based, we must model a mathematical measuring operator, as discussed. So why would the golden ratio obey the *principle of efficient language* better than any other ratio? Why would it be more powerful in terms of the ratio of symbolism to meaning?

For golden ratio divisions, going down from long to short, the ratios between successive pairs is the golden ratio. Going up, it is the inverse of the golden ratio. This quality is known as inflation and deflation and the golden ratio achieves it with only two symbols or numbers 1 and $1/\phi$.

But for divisions other than the golden ratio, going down needs two ratios ($3/2$, $2/1$ for example); going up also requires two ratios. Consider the idea that Planck scale measurement operators in the quantum gravity code use abstract *observation actions* in the E_8 derived

quasicrystalline point space to actualize compact symbolic objects that are themselves ratios – simply ordered arrays of the two Dirichlet integer values 1 and $1/\phi$. This binary pair of values is maximally efficient in terms of the symbolism to meaning ratio.



For the Φ (golden ratio) division, there is only one ratio needed for encoding the relationships of the consecutive segments going down in the length

$$\frac{a}{b} = \frac{b}{c} = \phi,$$

or going up in the length

$$\frac{c}{b} = \frac{b}{a} = \frac{1}{\phi}.$$

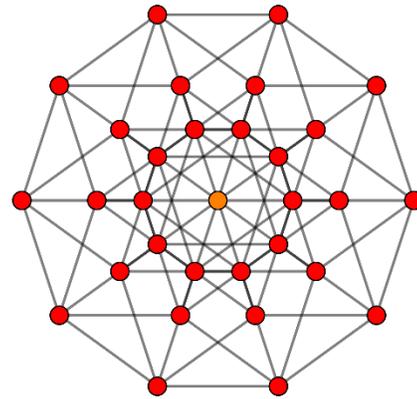
For example, any other division, such as dividing into thirds, requires two ratios and therefore more symbolic information to express:

$$\frac{a}{b} = \frac{3}{2}, \frac{b}{c} = \frac{2}{1} \text{ or } \frac{c}{b} = \frac{1}{2}, \frac{b}{a} = \frac{2}{3}.$$

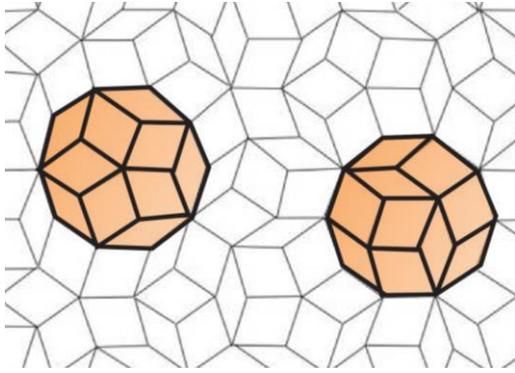
The other aspect of the golden ratio that is powerful and may be important for a simulation code of reality is its fractal nature. In the last 37 years, fractal mathematics has been found to be at play at all scales of the universe from cosmic to the sub-atomic scales (49). Dividing a line by the golden ratio, if we take the short length and place it on top of the long length, we are left with a section of the long length that is left over. That length is even shorter than the short length of the first division. And the ratio of this new short length to the original short length is the golden ratio. And this process can continue to infinity in the smaller direction with the ratio of the remainder to the previous length always being the golden ratio. Furthermore, this process can be applied in the other direction, where we add the long piece from the original division to the undivided length. The ratio of the new combined length to the long length from the first division is the golden ratio. This also continues to infinity. The golden ratio is the ultimate recursive fractal, generating the most information for the least amount of symbolic symbolism and *measurement action*.

A *phason flip* in a quasicrystal is a binary state change of a point, where it is registered as being *on* or *off*. If it

is *on*, it is an active node with a connection to other points in the quasicrystal. The syntax rules allow legal choices of whether a point can be *on* or *off*. One can call the total set of points the *possibility space*. The points that are chosen to be *on* by the code user, are active in that *frame* of the dynamic quasicrystal. Active or *on* points have connections and are syntactically legal selection configurations of the possibility space. For example, this is a projection of the 32 vertices of the 5-cube to the plane, where we see 31 total points with an overlapped 32nd point hidden in the middle. Note that the Penrose tiling is made by projecting a slice of the 5-cube lattice to the plane. These 31 points are a small section of the *possibility space* that the dynamical phason code of the Penrose tiling operates on.



The Penrose tiling is a tiling of two types of selection pattern of 16 of the 31 point decagonal *possibility space*. The two decagons can overlap other decagons in two ways or kiss without overlapping. Below, we highlight those two selection patterns, as they exist in a larger Penrose tiling quasicrystal. You can easily visualize how to select one of the two 16 point combinations below by looking at the above projection of the full 31 point *possibility space*.



An *empire* consists of the points that are forced to be *on* when you make a binary choice for a given point to be *on*. For example, look again at the first diagram of the 31 points. Note that if you select the center point to be *on*, you are forced to have a certain 15 additional points also be *on* and another 15 to be *off*.

Those 15 points that were forced to be *on* are the *empire* of the one point which you, the language user, chose to be *on* in a *phason flip* binary action.

Why is this interesting in terms of efficiency? In computer theory, we try to conserve binary actions. It costs electricity and time to open or close logic gates in a computer. So efficiency is important. We want codes that achieve maximal information with as few actions as possible.

Changing a single point to be *on* or *off* in a Fibonacci chain 1D quasicrystal forces an infinite number of additional points throughout the *possibility space* of the 1D chain to also change state. This global or non-local “spooky action at a distance” is very powerful in terms of the ratio of *action* to *meaning*.

If we live in an information theoretic universe, then, abstractly, the action we are trying to conserve is binary *choice*. Speculation of what the substance or entity or action is that makes the choices in the code is flexible. Our frame work will deal with the math and behavior of the code, not so much who or what the operator of the code must be. The *principle of efficient language* requires the operation of the code to tend toward maximal meaning for the least number of *on-off* choices.

Resources are always used to make to a choice in any physical model. For example, in the neural network of a human brain, choices cost calories and time. In an artificial neural network, choices require time and electricity. So efficient neural networks generate as much meaning as possible with a given number of connection actions – they generate maximal information for as few binary choices as possible.

Each Fibonacci chain is isomorphic to a *Fibonacci word*, which is a string of 0s and 1s that encodes a unique integer (51). Of course, the larger the integer, the greater the magnitude of the information. For example, a one millimeter Fibonacci chain with Planck length tiles is isomorphic to a *Fibonacci word* with 10^{31} 0s and 1s and corresponds to an equally enormous integer. Changing one point on the Fibonacci chain *possibility space* from *off* to *on*, changes the state of points along the entire chain, thereby changing the *Fibonacci word* to a different integer. The principle of *empires* in artificial neural networks consisting of networks of Fibonacci chains involves enormous efficiency when one is interested in conserving binary actions or choices. If nature is a computational language based on neural network theory and globally distributed computation and connectivity, quasicrystal codes are the most efficient possible.

When a network of Fibonacci chains is formed in 2D, 3D or 4D, a single binary state change at one node in the *possibility space* changes Fibonacci chains throughout the entire 1+n dimensional network of chains.

DNA AND QUANTUM COMPUTERS AS EXAMPLES OF 3D NEURAL NETWORKS

Manmade computer code symbols are, in some sense, minimally efficient. That is, the binary symbols encode an instruction to do one binary state action in a logic gate to and express only one bit of information. DNA is not quite a computer. It is a neural network in the sense that it both computes information using its code rules and it transfers information within its structure. Like a neural network, it achieves its computations and information storage in a distributed manner within 3-space. A single position in the DNA *possibility space* of coordinates where one of the four molecules in the code can exist serves as information in more than one

1D string of code. For example, an adenine molecule can exist at some location in the DNA coordinate space. This then forces certain states of the 4-letter code for other positions in the string, according to syntactical rules of the code. That string of code is wrapped around the double helix and has an *empire* of forced coordinate identities from the four letter code. But the *empire* is not just in 1D along that single string. Information relative to that one molecule selection of adenine is also encoded into strings that run in-line with the axis of the double helix and also diagonal to the axis. It is similar to the analogy of the game *Scrabble*, where a choice of a single letter on the grid of the possibility space of the game can encode information in more than one word. So the choice of the adenine molecule at that coordinate achieves a great deal of efficiency by (a) playing a role in multiple 1D strings and (b) by forcing other syntactically controlled actions of coordinates in the empire of that single registration of adenine in the DNA *possibility space*.

DNA has quasicrystalline structure. In fact, Erwin Schrodinger first deduced that DNA has a quasiperiodic structure in his book *What Is Life*, published nine years prior to Watson and Crick's discovery of DNA in 1953 (52). His deduction is in-line with the theme of this paper. Specifically, crystalline structures are deterministic and have no degrees of freedom in terms of their abstract construction. They are not inherently languages because they are too rigid in their construction rules.

On the other hand, amorphous or disorderly materials do not have structural rules and can have a virtually infinite number of microscopic geometric relationships – *geometric symbols*. The lack of rules and lack of a finite set of geometric symbols prevent a dynamic code from evolving within amorphous materials. The “sweet spot” between order and disorder, where a language or code can emerge, is in quasicrystalline order. Only within aperiodically ordered structure is there a true code with a finite set of geometric symbols, rules and syntactical freedom.

The most powerful codes are based on the golden ratio because the ratio of symbolism to geometric meaning output is maximal. For example, DNA is made of two helices that have pentagonal rotational symmetry,

which is based on the golden ratio. The two helices themselves are then offset from one another by a golden ratio related value called a Fibonacci ratio, which is a rational approximation of the irrational golden ratio (53).

Quantum computers are another example of systems where one node serves multiple roles in various relationships. 3D clusters of atoms, often with golden ratio based icosahedral symmetry (54), in a quantum correlated state interact with one another in various combinations to process and create information as a group – a spatial network of nodes very different than the ordinary notion of a 1D computational system.

SYMBOLIC POWER OF FIBONACCI CHAIN NETWORKS

It is well known that Fibonacci codes have unique and powerful properties in terms of error correction and detection (51).

For example, all sequences in a *Fibonacci word* end with “11”. And that sequence appears nowhere else in the data stream of that symbolic group object. Changing a bit corrupts the sequence (the symbolic group object). However, within a few more symbols, the pattern “11” will appear again, which indicates the end of the string or group symbol.

The system or user can then simply resume coding with only those few symbols left to be incorrect. The power in this is that one bit can only corrupt up to three symbols. No other code shares this property. Error detection is fast, and errors are limited in how much damage they can do. Error correction is similarly powerful and unique. Let us say that a 0 is erroneously changed to a 1 that is adjacent to a correct 1. A 1 that is part of the data stream gets changed to a 0. A 1 that is part of the ending 11 gets changed to a 0 and so on. When an error occurs in ordinary codes, it will exist uncorrected in the string forever.

The power of 3D networks of Fibonacci chains relates to the spatial dimension of the quasicrystal being able to host objects with icosahedral symmetry. For example, the 4D analogue of the icosahedron is the 600-cell (55). The icosahedron is one of the five regular polytopes in 3D – the *Platonic solids*. Three of

the solids correspond to crystal symmetries because their combinations can tile space. These are the square, octahedron and tetrahedron. The other two are correlated with quasicrystal symmetry, the 600-cell and the 120-cell. These correspond to the quasicrystal based Platonic solids called the icosahedron and dodecahedron, each possessing icosahedral symmetry. Again, in 3D there are five regular polytopes. In 4D, there are six. And in all dimensions higher than 4D there are only three – the analogues of the tetrahedron, octahedron and cube – the crystal related polytopes. The quasicrystal related regular polytopes are exclusive to dimensions less than 5D. So the special dimensions for Fibonacci chain related quasicrystals are 1D, 2D, 3D and 4D. And of these dimensions, 4D can host the quasicrystal with the densest network of Fibonacci chains, where 60 Fibonacci chains share a single point at the center of the 600-cells in the E_8 to 4D quasicrystal discovered by Elser and Sloane (43). In other words, a binary state change in the *possibility space* for this quasicrystal changes the state of many other Fibonacci chains associated with that point. And numerous other points in the possibility space also change state, not just the ones in the Fibonacci chains connected to the aforementioned point. All this binary state change – the *empire* – occurs due to the geometric first principles via the state change of a single node in the possibility space.

If the universe is a neural network interested in maximal efficiency, this would use a substrate like this. The fact that this quasicrystal and its 3D analogue discovered by our group called the *quasicrystalline spin network* encodes gauge symmetry unification physics may be evidence for the trueness of the conjecture. And this would be more likely if the universe is a neural network code concerned with expression of maximal meaning for the minimum number of binary state choices/actions.

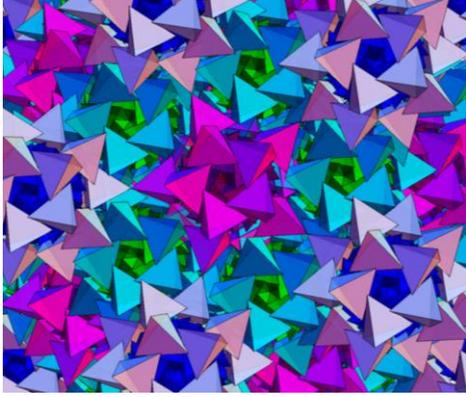
The *principle of efficient language* guides the behavior of the code choices in this framework, where binary actions in the code are chosen such that maximum information or meaning is generated for the least number of binary choices.

Meaning comes in two categories:

1. Physical or ultra-low subjectivity geometric information – the prototiles of the quasicrystalline code, wherein all particles and forces can be simulated such that the simulation are one and the same and are themselves physical reality.
2. Emergent or virtually transcendent and highly subjective information, such as mathematics and humor. This form of information can never be separated from the geometric physical information and quasicrystalline code. For example, the abstract thought of “love” comes with a package of memories and associations that trigger countless changes in the non-local waveform domain of quantum mechanics, gravity and electromagnetism.

At a physical level, evidence for this tendency toward efficient code use would exist in the form of the *principle of least action* and similar principles and conservation laws. At a non-physical level, evidence for this would exist in the form of the delayed choice quantum eraser experiment (56) and Bem’s retro-causality experiments (57) in addition to well known experiments of quantum entanglement over space and time. As engines of abstract meaning generation and perception, humans would be a special case in a universe obeying the *principle of efficient language*, where our perceptions of meaning and information far exceed the brute simple geometric meaning expressing physical phenomena in the quasicrystalline code.

The degree 120 vertices of the E_8 to 4D quasicrystal appear to be the maximum possible density of Fibonacci chains in a network of any dimension and therefore the most powerful possible *possibility space* for a neural network. 3D quasicrystals ordinarily have a maximum of degree 12 vertices with six shared Fibonacci chains. Fang Fang of Quantum Gravity Research discovered how to create a 3D network of Fibonacci chains with degree 60 vertices (15).



This quasicrystal is made entirely of 3-simplexes, the simplest possible “pixel” of information in 3D. And it encodes E_8 unification physics and is derived from the aforementioned E_8 to 4D quasicrystal. We are preparing a manuscript for publication that may prove the maximum density and efficiency claims made herein for both the 3D and 4D quasicrystals.

IS THE ERROR CORRECTION CODE FOUND IN GAUGE SYMMETRY PHYSICS A CLUE THAT NATURE COMPUTES ITSELF INTO EXISTENCE?

James Gates, the John S. Toll Professor of Physics at the University of Maryland and the Director of The Center for String and Particle Theory found the widely used *doubly-even self-dual linear binary error-correcting block code* embedded in the network of relationships of the gauge symmetry unification equations of fundamental particles (58). These are the exact same codes used in web-browsers and peer-to-peer network simulations to ensure the consistency of information transfer from client to client. Furthermore, he found that the error correction codes relate specifically to geometric symbols, called *adinkas* (59), which encode the relationship of particle gauge symmetry equations. The astounding finding is one of the most powerful pieces of evidence in support of the *digital physics* view that is growing in popularity in academic circles – the view that reality itself is a computation, essentially a simulation (60). Gates himself commented, “We have no idea what these things are doing there” (61).

IS THERE EVIDENCE FOR A GOLDEN RATIO CODE IN BLACK HOLE EQUATIONS, QUANTUM EXPERIMENTS AND SOLID STATE MATTER?

Other compelling evidence used to support the *digital physics* view includes black hole quantum gravity theory and an idea known as the *holographic principle*, which is derived from the mathematical proof called the *Maldacena conjecture* (62). It states that the total amount of binary information from all the mass and energy pulled into a black hole is proportional to its surface area, where every four Planck areas of its surface encodes the state of a fundamental particle that fell into it.

It is a distinctly binary code based framework that comes directly from the first principles application of general relativity and quantum mechanics at the limit of both theories – the environment of a black hole. Black hole quantum gravity equations are one of the best clues we have about what a theory of everything might look like.

As stated, quasicrystals generally relate to five-fold symmetry and the golden ratio. The pentagon is the 2D analogue of the icosahedron and the quintessential 2D quasicrystal is the Penrose tiling with its 5-fold symmetry related golden ratio structure. Virtually all of the 3D quasicrystals discovered in nature have icosahedral symmetry. That symmetry is possessed by any object having the combination of 2-fold, 3-fold and 5-fold rotational symmetry. The E_8 to 4D quasicrystal has these symmetries and is fundamentally based on the golden ratio.

Black hole physics relates deeply to the golden ratio. It is the precise point where a black hole’s modified specific heat changes from positive to negative (63).

$$\frac{M^4}{J^2} = \phi$$

And it is part of the equation for the lower bound on black hole entropy.

$$e^{\frac{8\pi S l_P^2}{kA}} \geq \phi$$

The golden ratio even relates the loop quantum gravity parameter to black hole entropy (64).

$$2^{\pi\gamma} = \phi$$

In 1993, Lucien Hardy, of the Perimeter Institute for Theoretical Physics, discovered that the probability of entanglement for two particles projected in tandem is (65):

$$\phi^{-5}$$

In 2010, a multinational team of scientists found an E_8 based golden ratio signature in solid state matter. Cobalt niobate was put into a quantum-critical state and tuned to an optimal level by adjusting the magnetic fields around it. In describing the process, the researchers used the analogy of tuning a guitar string. They found the perfect tuning when the resonance to pitch is in a golden ratio based value specifically related to the geometry of E_8 . The authors speculated that the result is evidence in support of an E_8 based theory of everything (66).

Xu and Zhong's short paper (67), *Golden Ratio in Quantum Mechanics*, points out the connections to the golden ratio in various works – linking it to particle physics and quantum gravity (quantized spacetime). The short piece is worth reprinting here, and we have included their citations in our bibliography.

The experimental discovery of the golden ratio in quantum magnetism (68) is an extremely important milestone in the quest for the understanding of quantum mechanics and E-infinity theory. We full-heartedly agree with the explanation and discussion given by Prof. Affleck (69)... ...For this reason we would like to draw attention to a general theory dealing with the noncommutativity and the fine structure of spacetime which comes to similar conclusions and sweeping generalizations about the important role which the golden ratio must play in quantum and high energy physics. Maybe the most elementary way to explain this point of view is the following: Magnetism is just one aspect of the five fundamental forces of nature. In a unified picture where all the five forces melt into one, it is reasonable to suspect that the golden ratio will play a fundamental

role. This fact immediately follows from the work of the French mathematician Alain Connes and the Egyptian engineering scientist and theoretical physicist M.S. El Naschie. In Connes' noncommutative geometry his dimensional function is explicitly dependant on the golden mean. Similarly the bijection formula in the work of El Naschie is identical with this dimensional function and implies the existence of random Cantor sets with golden mean Hausdorff dimension as the building blocks of a spacetime which is a Cantor set-like fractal in infinite dimensional but hierarchal space. Invoking Albert Einstein's ideas connecting spacetime to geometry with energy and matter, it is clear that these golden mean ratios must appear again in the mass spectrum of elementary particles and other constants of nature. There are several places where this work can be found (70) (71) (72).

WIGNER'S UNIVERSALITY

The *universality* pattern is another fundamental clue about what a theory of everything should look like. It is aperiodic but ordered – liberally defined as a quasicrystal. It was first discovered by Eugene Wigner in the 1950s in the energy spectrum of the uranium nucleus (73).

In 1972, number theorist Hugh Montgomery found it in the zeros of the Riemann zeta function, so it deeply ties into the distribution of prime numbers (11). In 2000, Krbálek and Šeba reported it in the complex data patterns of the Cuernavaca bus system (74). It appears in the spectral measurements of materials such as sea ice (75) and human bones. In fact, it appears in all complex correlated system – virtually every physical system. *Wigner's hypothesis* states that the universality signature exists in all complex correlated systems (9). Van Vu of Yale University, who has proven with coauthor Terence Tao that *universality* exists in a broad class of random matrices, said, “It seems to be a law of nature” (12).

Why something as fundamental as the *universality* signature would relate to both the distribution of primes and complex physical systems is a mystery – unless somehow number theory and an unknown theory of everything are deeply related. Of course, that is trivially true since the entire edifice of mathematics is built upon the counting numbers. And the

foundational “skeleton” of the counting numbers are the primes. Eugene Wigner famously said that nature is unreasonably mathematical (76). So the ultimate foundation of both complex mathematics and nature herself reside in number theory.

Freeman Dyson defines a quasicrystal as “a [aperiodic] pure point distribution that has a pure point spectrum”. He said, “If the Riemann hypothesis is true, then the zeros of the zeta-function form a one-dimensional quasicrystal...” (2). Andrew Odlyzko published the Fourier transform of the zeta-function zeros. It showed sharp peaks at the logarithm of the primes and prime (10). This demonstrated that the distribution is not random but is aperiodically ordered. By the same definition, the *universality* signature is a quasicrystal. Quasicrystals in nature generally correspond to the golden ratio. So how might the *universality* signature correspond to it?

Universality relates fundamentally to matrix math. It defines the spacing between the eigenvalues of large matrices filled with random numbers. This is interesting because the four-term two-by-two binary matrix is the most fundamental of all matrices. 14 of its 16 possible combinations of 1 and 0 have either trivial or simple eigenvalues as 0, 1 or 2. However, the remaining two eigenvalues are golden ratio based as:

$$\lambda_+ = \phi \text{ and } \lambda_- = -\frac{1}{\phi}$$

Quantum systems, such as the hydrogen atom, are governed by matrix mathematics. Freeman Dyson said, “Every quantum system is governed by a matrix representing the total energy of the system, and the eigenvalues of the matrix are the energy levels of the quantum system.”

Based on work done by Suresh and Koga in 2001 (77), Heyrovska (78) showed the atomic radius of hydrogen in methane to be the Bohr radius over the golden ratio.

$$r_H = \frac{a_0}{\phi}$$

The random matrix correspondence to physics is not an indication that actual randomness occurs. The matrices

of some correlated systems, like a hydrogen atom, can be worked out precisely. However, more complicated systems, such as a uranium atom, are non-computable by current methods. The values of its unknown matrix become super-imposed like the blur of voices in a crowded conference hall. Although, there is no randomness in the conversations of the people in the crowd, the super-position of soundwaves behaves exactly like the solutions to a matrix with random numbers.

Scientists are still trying to figure out why *universality* has the exact pattern that it does. Vu said, “We only know it from calculations”. Because this pattern also matches perfectly to the distribution of the non-trivial zeros in the Riemann zeta function, the distribution of primes must relate to a strongly correlated matrix. Dynamically, quasicrystals obey random matrix statistics (79). And they are strongly correlated and non-local, due to the *empire* concept discussed above.

The distribution of prime numbers is encoded in the spectral pattern derived by an irrational projection of a slice of an A_n lattice to 1D. The cell types of A_n lattices are simplex-integers, n-simplexes, where each A_n lattice and its n-simplex cell type embeds the stack of all A_n lattices with dimensions lower than it. That is, the series of simplex-integers, including prime-simplex-integers, are encoded in the projection of a slice of an A-lattice to 1D. This will be discussed in more detail in Part III.

The salient point for now is that the distribution of primes and, accordingly, the zeta zeros corresponds to geometric-number theory – simplex-integers and their associated A_n lattices. We conjecture that our quantum gravity framework based on a quasicrystal projected from the 8-simplex based E_8 lattice will explain why the quasiperiodic *universality* pattern appears both in nature and prime number theory. That is, the matrix analogue of our quasicrystal may be the missing matrix correlated to the *universality* signature.

Like, all quasicrystals, the dynamical behavior of our E_8 derived quasicrystal is described by a complex matrix (79). Because its complex phason code is strongly and non-locally correlated, it will obey random matrix statistics and map to the *universality* signature. But the random matrix and universality pattern would be secondary. We agree with László Erdős of the University of Munich, who said “It may happen that it is not a matrix that lies at the core of both Wigner’s *universality* and the zeta function, but some other, yet undiscovered, mathematical structure... ..Wigner matrices and zeta functions may

then just be different representations of this structure” (80).

PART IV CONCLUSION

This section began with the conjecture:

If nature were a self-organized simulation, it would be a simplex-integer based quasicrystalline code derived from E_8 .

We have defended the reasonableness of the conjecture. Now it is up to our institute and the scientists who work here to publish a series of theoretical and experimental papers that transform the toy framework into a rigorous formalism worthy of attracting a community of collaborators. The approach is certainly outside the box. However, an outside the box approach may be what is needed. String theory is now 50 years old and it has not made a successful prediction. We believe that a fresh but rigorous new approach such as ours is overdue. It is possible there are bridges to aspects of the string theory approach. In fact, the most foundational string theory was first introduced by David Gross et al in 1985, *heterotic string theory*. It exploits the power of E_8 .

However, our primary approach, achieves symmetry breaking in an intuitive manner via projective geometry to lower dimensions, where full recovery of hyper-dimensional unification physics can be achieved. The resulting spacetime and particle code is a simulation, much more similar in form to *loop quantum gravity*, where the code itself is the structure of dynamical spacetime.

Appendix – Overview of Emergence Theory

Emergence theory, developed by our institute over the last eight years, exploits the ideas discussed above. The program is at an intermediary stage of development.

Foundational Papers – Fang, Sadler et al published the foundational tetrahedral golden ratio rotational relationships and helical behavior in 2013 (81). In 2012 Kovacs et al introduced the *sum of squares law* (82) and in 2013 Castro-Pearlman et al proved the derivative *sum of areas and volumes law* (83). In 2014, Fang et al derived the golden ratio rotation from

the first principles approach of the *icosagrid* method. In 2016, she and coauthors published the construction rules of the 3-simplex based quasicrystalline possibility space and introduced the term *golden matrix* (“GM”) to describe it along with its E_8 derived sub-spaces (15).

Conceptual Overview – Our program is an *Occam’s razor* approach to physics, where we aim to start with irreducible first principles and relentlessly question status quo assumptions. Because nature seems to be governed by rules and beautiful math, it is safer to say that there exists an analytical expression for the fine structure constant, the Planck constant, the magnitude of the speed of light and the gravitational constant than it is to say there is not. Put differently, either there exists a first principles theory of everything that explains these values or there is not. However, no such theory has been discovered yet. All theories start with those values and then create equations relating them and their composite objects.

It is helpful to understand the difference between a *unification theory* and a *simulation theory*. A *unification theory* is a network of equations that show how different things transform into one another. A *simulation theory* uses geometric building blocks as the mathematical operators that themselves *are* physical reality – the simulation – instead of merely describing it. Such a framework would spit-out the unification equations while also serving as the “pixels” or functional building blocks of reality. We want to know what reality is, not just the equations that tell us how it behaves or how it is unified. *Loop quantum gravity* is the most popular simulation theory.

Because reality appears to have three spatial dimensions, we start there and inquire whether or not it is possible to simulate physics using the simplest building block or pixel of 3D information, the 3-simplex. The idea is known as a *background independent* model because it starts with spacetime building blocks and makes particles the propagating patterns in that system. The second part of our basic idea is that we use a quantized irreducible unit of measurement at the Planck scale substructure of our model. We call this operator a *quantum viewer*. The building block simplex-integers are made of information. But, they are ontologically real because

they are being actualized by quantized units of primitive measurement – the *quantum viewers*.

We will now highlight a few of the key components of our framework.

Ontology and symbolic language – “All that exists is that which is measured.”

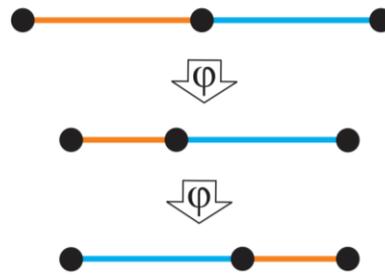
We would agree with Ilija Barukčić’s statement:

“Roughly speaking, according to Bell’s theorem, *there is no reality separate from its observation*” (84).

Classic physics indirectly defines energy as information in the form of an abstract quantity called the “potential for work”. Spacetime is permeated with energy, where different energetic potentials within it different densities of curvature. Einstein’s mass-energy equivalency reduces matter to the notion of bound up energy (85). Quantum mechanics is more clearly information theoretic, dividing reality into the abstract possibility space of the wave function and the actualized collapse into a particle coordinate in the form of measurement data (86). John A. Wheeler was one of the first to point out that reality is made of information (35). Max Tegmark and many other modern physicists hold this view today. Information is real, so ontologically, there is a divide between the potential for information, which is not real, and information as a product of measurement/observation, that is real (87). The *measurement problem* associated with quantum mechanics relates in large part to the choice of ontological interpretations of what the equations and experiments mean. It is a topic of hot debate with no broad consensus. Einstein and many others have said that there is something we are missing and that the formalism is incomplete (88). Some have taken the bold position that humans or entities at our level must measure something to actualize it into physical existence. Einstein was one of the first to take issue with this idea, saying, “I like to think that the Moon is still there even when I’m not looking at it”. So we take the more conservative position that there is some self-actualizing measurement operator at the Planck scale, where the quantized *pixels* of reality exist. We call this operator a “*quantum viewer*”. Its function is to generate a trinary state change in the 3-simplex quanta of space in a *possibility space* of such

objects. The possibility space is called the *quasicrystalline spin network* (“QSN”) (15). It is an E_8 derived space of 3-simplexes, wherein the trinary state selection actions create syntactically legal quasicrystalline subspaces of the QSN that are physically real frames of space with particle patterns embedded within it. The trinary quality state choices are: (1) on right, (2) on left and (3) off. For example, if a 3-simplex is in the “on right” state in one quasicrystalline frame and is “on left” in the next frame of a dynamical sequence, the formal action is a Clifford rotor or spin operation on the possibility space. However, there is an ontological requirement to manifest these actions with an irreducible measurement/observation operator – the *quantum viewer* action. To understand this, visualize the idea of standing to the left of a friend and taking a photo. Next, walk to her right and take a second photo. Each photo is a transformation-symbol. And the ordered set of two photos express the physical information of a discretized rotation of your friend changing orientation relative to your camera if you stationary and she rotated between the two orientations. So as each *quantum viewer* performs its operation, it captures symbols which are projective transformations that are equal to a state change of a tetrahedron as either on-right, on-left or off. The *quantum viewers* actualize, via observation/measurement, the action of a Clifford rotor or spin operation on the *quasicrystalline spin network*.

As mentioned above in the section titled *Quasicrystals as Maximally Efficient Codes*, the 3-simplex network can be decomposed as a network of 1D Fibonacci chains with line segments in the golden ratio proportion. The *quantum viewers* generate either a right or left handed rotation of a tetrahedron, which divides a given edge by the golden ratio on one side or the other.



Mathematically, the coordinates of the *quantum viewers* – the camera positions – are the edge crossing point set of the *quasicrystalline spin network*. A key geometry of the network can be understood by taking twenty evenly spaced 3-simplexes that share a common vertex at the center of the cluster. Rotating each either right or left on an axis running from the outside face center through the shared inner vertex by the golden ratio based angle $\frac{1}{2} \left(\cos^{-1} \left(\frac{1}{4} \right) - 120 \right)^\circ$ creates a 20-group that is either twisted to the right or left. This is an absolute chirality not relative to one's vantage point.



In summary:

1. The *quantum viewers* are the observation or measurement operators.
2. They make projective transformations based on their position, just as a camera transforms a 3D image to a 2D image, which is really a network of transformations of 1D actual lengths (lines) between pairs of points to contracted or transformed lines. So the irreducible measurements are 1D phason flips that divide a line into the golden ratio with the long side on one side of the line or the other.
3. The transformations are information. They are observations that are equal to symbols. Because those symbols are ontologically real due to actualization via observation, they compose the next frame or state change in that region of the QSN – a physically real region of space and time with particle patterns in it.
4. Formally, the system is a spin network on a discretized moduli space, where the operators are primitive measuring entities generating physically real information.
5. Its rules and syntactical degrees of freedom are derived by the geometric first principles of *phason cut+projection* dynamics related to the movement of a *cut window* through the Elser and Sloan E_8 to 4D quasicrystal.

Quantized Space and “Time”

As explained above, space is quantized as 3-simplexes. And time is quantized like a 35 mm film, as ordered sets of individual quasicrystalline frames of 3-simplexes generated by ordered sets of trinary selection choices of the *quantum viewers* in the QSN. Of course, this concept of a universal frame rate is anathema to key assumptions in special relativity – the invariance of the speed of light and the notion of smooth spacetime. The old relativistic notion is that, because spacetime is smooth and structureless, nothing can have intrinsic time or motion but only relative time and motion. The relativistic concept is well supported by experiments, which show that, no matter how fast an observer is chasing a photon, it always seems to elude him at the speed of light. Our solution to this is the electron clock model.

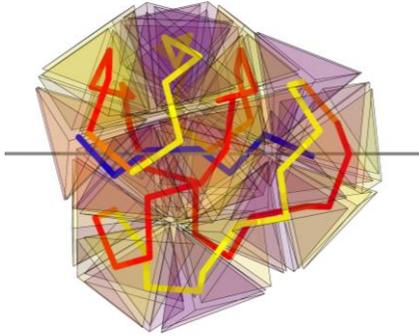
Electron Clock Intrinsic Time

We reject the assumption of structureless space. The Michelson-Morley experiment of 1887 was not designed to test for a structure as described herein or any of the other *loop quantum gravity* type theory, where spacetime has a discrete substructure. Prior to 1887, the scientific community presumed a specific fluid type material called the *aether* filled space (89). When experiment did not demonstrate this substance, a new axiom was established – that there is no substructure to space. Of course, without substructure, there can be no logical motion relative to space. An object would not have intrinsic motion but only motion relative to another object floating in the ocean of the structureless vacuum. This key axiom undergirds relativity theory. The second modern assumption is that fundamental particles, like the electron, have no substructure and are instead dimensionless points. If were true, such a particle could not have an internal clock or any concept of rotation. All time or *change* that would be ontologically real would be change relative to another object changing – another clock.

Louis de Broglie first conceptualized the notion of the electron possessing an internal clock (90). Later David Hestenes made this idea more rigorous (91). In the *emergence theory* framework, massive particles, like an electrons are composites of multiple Planck length

3-simplexes chosen as ordered sets in frames of the QSN. There are two forms of dynamic pattern:

1. *Stepwise toroidal knot* – This is a knot pattern much like a 3D trefoil knot that has an asymmetric region that cycles around the geometry of the knot. Multiple quasicrystal frames are required in order to complete a full cycle around the knot – a *tick* of the internal electron clock.



2. *Helical propagation* – For simplicity, let us imagine it takes 10 frozen quasicrystal states chosen by the *quantum viewers* to compose an animation of one knot cycle. The entire knot can remain at one coordinate in the QSN or it can propagate helically forward in a certain direction. However, if any of the 10 frames are used to propagate the pattern forward, there will be fewer frames available to complete time cycles of the internal toroidal knot-like clock. There must always be a rational fraction of frames used for propagation and frames used for clock time. The two patterns of “time” and propagation would be inversely proportional to one another. And there would always be an absolute and intrinsic ratio of internal clock-time to propagation with respect to the global frame rate of the QSN.

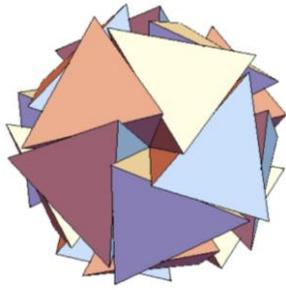
A photon in this model is a pattern of tetrahedra in the QSN that is only helical, not toroidal. So the ratio of propagation to clock time in a given number of frames will always be 100:0. That is, any non-massive particle (particles without internal knot structure) will always propagate in an invariant manner with the same distance covered over a given quantity of frames.

The traveler in a spacecraft moving at 99% of the speed of light will shift their intrinsic clock cycles (as a ratio of total frames) to a very slow rate. This will

include all massive particles moving with it, including the measurement apparatus and the operation of the brains of the scientists onboard the craft. The clock cycles or experience of change on the craft will be very slow and the photon will move at the speed of light from the projector on the ship and will go to a mirror at some distance before reflecting back to the measurement apparatus to be compared to some quantity of clock cycles. Very few clock cycles will have elapsed because time for these travelers and their massive equipment will slow to a near halt. Accordingly, the comparison of the distance traveled by the photon to the number of clock cycles will indicate that the photon moved relative to the traveling craft at the same speed it moved when the experiment was done while the vehicle was moving at 1% of the photon’s rate of propagation. However the intrinsic or actual difference between the speed of the vehicle moving at 99% of the speed of light and photon moving at 100%, would in truth be 1% of the speed of light. Clearly, this viewpoint is far less enigmatic and geometrically pleasing than the ordinary interpretation of these experiments via the smooth spacetime ontology of special relativity.

Chirality

The conjecture that fundamental particles are dimensionless points without structure causes intuitive geometric confusion with other indications that particles deeply relate to handedness or chirality. For example, a current of electrons has a well understood geometric chirality feature. The right handed rule of how a magnetic field is wrapped around the current in a chiral fashion tells us something deep about handedness in nature. However, the notion of a right handed or left handed individual particle is replaced by an abstract non-geometric sign value that is distinctly non-geometric due to the conjecture of the dimensionless point particle identity of the particle. For example, the point particle mathematical abstraction is one where helicity is the sign of the projection of the spin vector onto the momentum vector, where left is negative and right is positive. It is an outstanding mystery as to why the *weak interaction* acts only on left-handed fermions such as the positron and not right-handed ones like the electron (92).



Quasiparticle patterns in the CSN have a fundamentally different feature that relates to chirality. The above image is left-handed group of twenty 3-simplexes, where the states of the tetrahedra by the quantum viewers on the simplexes are all “on-left”.

Ordinarily, a helix made of 3-simplexes, as shown on the left, will have no periodicity because of the irrationality of the dihedral angle. However, in the CSN, tetrahedra can only be related by the golden ratio based angle based angle:

$$\frac{1}{2} \left(\cos^{-1} \left(\frac{1}{4} \right) - 120 \right)^\circ = \text{ArcCos} \left[\frac{\phi^2}{2\sqrt{2}} \right].$$



The helix on the left is right handed. So when the rotation of the golden ratio angle is of opposite chirality, in this case rotated by that value – right handed – the periodicity become 5-fold. And when it is rotated left, it becomes 3-periodic. The deep reason for these two periodicities corresponds the E_8 to 4D Elser-Sloan quasicrystal, wherein the projection of the Gosset polytopes in the E_8 crystal generates 600-cell made of 600 3-simplexes. Each simplex is part of a rings of 30 simplexes, as shown in this diagram.



The periodicities of the tetrahedra in this ring are a superposition of 3-fold and 5-fold, where the orientation of 15 tetrahedra repeats 3-periodically and 15 repeat 5-periodically. The dihedral angle between any two adjacent tetrahedra is $\frac{1}{2} \left(\cos^{-1} \left(\frac{1}{4} \right) - 120 \right)^\circ + 60^\circ$. In 4D, there is vectorial freedom for the 60° component of the angle. When the relationships of 3-simplexes are represented in the CSN, we cast out the 60° component because it is the portion related to the construction of a simplex series, where each 60° of a new edge on an n-simplex to generate an n+1 simplex is 60° into an additional spatial dimension. So there is

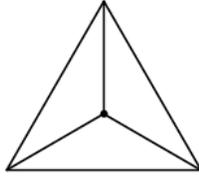
Realistic physics would not be able to be done if we projected the $E_8 \rightarrow 4D$ to 3D or projected E_8 directly to 3D. The key feature of the CSN is that, by making the tetrahedra regular by taking a 3D slice of the 4D QC with regular tetrahedra and then rotating copies of that slice by the same angle that relates adjacent tetrahedra in the 4D QC but minus the 60° component, we introduce three crucial elements into the object:

1. It generates an additional sign value necessary for physics.
2. It significantly increases the degrees of freedom in the code. In other words, it transforms the code from a binary on/off code to a trinary code of “on right”, “on left” and “off” in terms of the registration possibilities for a given tetrahedron in a frame of the QSN.
3. It improve the ratio of symbolism to meaning by reducing all of the tetrahedra to the simplest possible 3D pixel of information, the 3-simplex. If the 3D QSN were generated by projecting the E_8 lattice or the 4D QC to 3D, it would generate seven different shapes of distorted 3-simplexes. It would change the ratio of symbol simplicity rank to meaning in the code (see Part I).

Conservation and the Sum of Squares Law

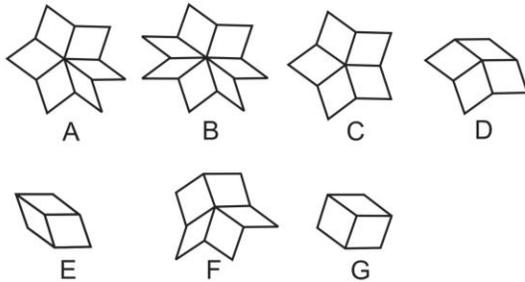
Conservation is an inherent quality of irrational projection based geometry. For example, consider a tetrahedron with four lines running from the centroid to each vertex. Assuming the edge length of the tetrahedron is one, we can project the four inner lines

to the plane with an infinite number of projection angles, such as in in projection below.



The sum of squares of each contracted length in the projection is always conserved as 4 or the integer corresponding to the simplex-integer, in this case the 3-simplex corresponding to the integer 4. The sum correlates in a mysterious way to the spatial dimension of a projected polytope, as reported in our two papers *The Sum of Squares Law* and *The sum of the squares of areas, volumes and hypervolumes of regular polytopes from Clifford polyvectors*.

Based on this same conservation principle, the “letters” or geometric symbol types of a quasicrystal are conserved. For example, there are seven different vertex geometries in a Penrose tiling, as shown.



Each of their frequencies of occurrence are conserved as follows:

- A = 1
- B = ϕ
- C = ϕ
- D = ϕ^2
- E = ϕ^3
- F = ϕ^4
- G = ϕ^5

Similarly, the various legal particle configuration patterns made of relationships between 3-simplexes chosen on the QSN have conserved quantities. We suggest that the deep first principles based explanation for Noether’s first theorem, gauge symmetries and conservation laws in nature is hyperdimensional

projective geometry, where the full encoding and richness of hyperdimensional structure is transformed into lower dimensional geometric symbolic code – quasicrystal language.

Quasicrystals have the fractal quality that any shape, such as the 7 vertex geometries in the Penrose tiling, repeat according to a scaling algorithm, typically the power series of the golden ratio, $\phi, \phi^2, \phi^3 \dots$

Alternative Expression of Geometric Frustration

The term, geometric frustration, can be thought of as “trans-dimensional pressure” resulting from a projection of an object to a lower dimension. For example, in 3D, there is vectorial freedom or space for 12 unit length edges to be related by 90° in the form of a cube. When projected along an irrational angle to 2D, the reduction of vectorial freedom compresses or transforms the information into a 2D representation that requires edges to contract and angles to change. The 2D projection or *shape-symbol* is a map encoding (1) the information of the pre-projected object and (2) the angular relationship of the projection space to the pre-projected object. The transdimensional tension or pressure is immediately released or transformed into the transformed lengths and angles of the projection.

An alternative form of transformation or transdimensional pressure expression is rotation and translation. For example, consider the transformation of a 20-group of tetrahedra sharing a common vertex in the 600-cell in 4D space. If we project it to 3D along a certain angle, we can generate a group of 20 distorted tetrahedra with a convex hull of a regular icosahedron

and 12 inner edge lengths contracted by $\frac{\sqrt{\phi\sqrt{5}}}{2} = \text{Cos}(18)$.

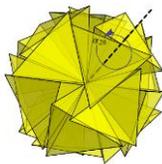
We can understand the difference of dimensions as a curvature of one dimension into a higher dimension. For example, a flat piece of paper can be curved into the 2nd dimension such it is a curved 2D object that requires three spatial dimensions to exist in.

So if we take our 4D 20-group, we can realize that it is bounded by a 3-sphere (4D sphere), which is a curved 2-sphere (ordinary sphere). And we can slowly de-curve or flatten the 3-sphere of space containing the 20-group until it is “flat”, at which point it is an

ordinary 2-sphere. In this case, the 20 regular tetrahedra living in 4D that have unit edge lengths would need to distort such that the 12 shared inner edges contract to $\text{Cos}(18)$. This result is identical to the aforementioned projection of the 4D object to 3D. An alternative method of encoding the projection or uncurving action is to anchor the 20 tetrahedra around their common shared vertex and rigidify them, such that they are not allowed to encode the geometric frustration via edge contraction and angle change. This will force the tetrahedra to express the information of their hyperdimensional relationships in lower dimensional space by rotating along each of their 3-fold axes of symmetry that run through face centers to opposing vertices (the shared center vertex).

Each of the 20 tetrahedra in the 4D space lives in a different 3D space related to the adjacent 3D space by $\text{ArcCos}([3\phi - 1]/4)^\circ + 60^\circ = \text{ArcCos}(1/4)^\circ$. If we visualize this as a gradual uncurving of the 4D space toward flat 3D space, we begin with zero rotation of each tetrahedron.

As we initialize the uncurving, the faces will begin to rotate from one another such that their 12 shared inner edges “blossom” into 60 unshared inner edges. As we do this, we are gradually intersecting or converging the twenty separate 3D spaces into a single 3D space. At the point where the 3-sphere bounding space is completely flattened to an ordinary 3D sphere, the rotation value between the kissing inner faces of the 20-group is $\text{ArcCos}([3\phi - 1]/4)^\circ$, which is the angular relationship between kissing 3D spaces containing tetrahedra in the 4D space of the 600-cell, minus the 60° component that there is no room for in 3D [see next section sub-section, *60° Construction*].



Now, we have a curvature value of 0 and a rotational value of $\text{ArcCos}([3\phi - 1]/4)^\circ$ and have encoded the relationships of the 20 tetrahedra living in 4D into a geometric symbol in 3D via rotation instead of edge contraction. We have converged 20 tetrahedra from 20 individual 3D spaces related to the other by

$\text{ArcCos}([3\phi - 1]/4)^\circ + 60^\circ$ into a single 3D space where they related by the same fundamental irrational component of their former relationships but without the 60° component that was used to construct them by extruding successive spatial dimensions by a process of rotating edge copies into the next spatial dimension by 60° .

We can now reverse the process and slowly curve the flattened 4D object that is now a 3D object back into a perfect 4D 20-group. As we do, the rotational value decreases and the space curvature value increases. At the point in which the 2-sphere and its rigid tetrahedra are curved into a perfect 3-sphere, the rotational value is 0 and the curvature value is $1/\phi$. So there is an inverse proportionality between the curvature limit and the rotational limit as 0 rotation $\rightarrow 1/\phi$ spatial curvature and $1/\phi$ spatial curvature $\rightarrow 0$ rotation.

In the QSN, every adjacent tetrahedral relationship is $\pm \text{ArcCos}([3\phi - 1]/4)^\circ$, which is the 4D angular relationship between tetrahedra in the E_8 to 4D quasicrystal minus the 60° component not related to 3D.

This special non-arbitrary rotational value is powerful for modeling quantum gravity and particle patterns for four reasons:

1. It encodes the relationships of tetrahedra in a 4D space, which can be useful for modeling 4D spacetime in three spatial dimensions.
2. It encodes the relationships of tetrahedra in an E_8 derived quasicrystal, which can be useful to model gauge symmetry unification of gravity and the *standard model* particles and forces.
3. It introduces a binary sign value, chirality. The edge distortion method of encoding geometric frustration does not generate the chirality value. This may be useful for fundamental physics, which uses three binary sign values (1) polarity, (2) spin and (3) charge.
4. The chirality sign value servers as an important degree of freedom in the quasicrystalline code, as opposed to a more restrictive ordinary quasicrystalline. This degree of syntactical

freedom makes the geometric language more powerful.

This fundamental rotational value is the basis of action on the QSN. That is, the Clifford rotor spin operations are this rotation, which will serve as the new \hbar , the reduced Plank constant or Dirac constant, in our emerging geometric first principles approach to fundamental physics.

There are two ways to visualize operations on the QSN:

Graph theoretically – The QSN is a network of points and connections (edges). It is simply an extended construction of the 20-twist discussed below. The 180 possible connections on the 60 points derived via any of the construction methods discussed are part of the possibility space. So graph theoretically, we can picture the 180 connections as a graph diagram in 3-space. And then we can do graph operations to turn edges “on” or “off” in order to make patterns.

Trinary code – Or we can turn entire tetrahedra “on” or “off” in which case we can think of a centroid of a tetrahedron as being selected and designated as either the right rotated or left rotated version or not on at all, for a total of three possible choices.

Clifford rotor/spin network – Or we can conceptualize the tetrahedra to rotate smoothly in a classical sense, such that it is rotated from a left to a right position via the $\text{ArcCos}([3\phi - 1]/4)^\circ$ rotation value. We can further decompose these rotations into individual edge rotations.

Simplex Construction by 60°

As mentioned in Part III, the simplex series is constructed by starting with an edge, a 1-simplex, and rotating a copy on a vertex by 60° into the next spatial dimension to form a 2-simplex or three equidistant points on the plane. A copy of one of those edges is then rotated by 60° into the 3rd spatial dimension to form an equidistant relationship of four points and a dihedral angle of $\text{ArcCos}(1/3)$. The dihedral angle series ranges from 60° in the 2-simplex to 90° in the infinite-simplex, spanning a total of 30° and where each dihedral angle in the series between 30° and 90° is

irrational as the ArcCos of a successive fraction from the harmonic series $1/2, 1/3, 1/4\dots$

We can think of the 60° component of each dihedral angle as being tied to the action that extruded an additional spatial dimension necessary for the next point to be added in such a manner that all points are equidistant. The remaining irrational component of each dihedral angle is the more “meaningful” part, carrying the key information of the given simplex-integer. For example, in the case of the 4D simplex, the two parts of its $\text{ArcCos}(1/4)^\circ$ dihedral angle are $\text{ArcCos}([3\phi - 1]/4)^\circ \approx 15.522^\circ$ and the 60° component correlated to the extra-spatial rotation that extruded out the next spatial dimension in the buildout process from 3D to 4D. The relationship between kissing 3-simplexes in a 4D space is $60^\circ + 15.522^\circ$. Accordingly, when one uses the irrational component of this angle in a 3D construction of regular tetrahedra, such as in our approach, it encodes the relational information between tetrahedra as they would have existed in, for example, the 4D Elser-Sloan quasicrystal derived from E_8 .

And because 15.522° is inversely proportional to the $1/\phi$ curvature value, as explained above, it is most deeply a transformation of the information of a finite 4D spaces (a 3-sphere of radius 1) into a finite 3D space – a 2-sphere of radius $\text{Cos}(18)$.

This same construction approach can also be used to buildout the E_8 lattice, which is a packing of 8-simplexes that leaves interstitial gaps in the shape of 8D orthoplexes.

Specialness of 3D and 4D

In 2D there are an infinite number of regular polytopes. But they all have rational angles and are trivial in some sense except for the ones based on the angles $60^\circ, 72^\circ$ and 90° as the equilateral triangle, pentagon and square. These are the polytopes corresponding to the five Platonic solids, the only regular polytopes in 3D. For example, the equilateral triangle is the polytope in 2D corresponding to the tetrahedron. Only the equilateral triangle and square can tile the plane, making them the “crystal” based 2D analogues of the Platonic solids. The pentagon cannot tile the plane and corresponds to the icosahedron and dodecahedron. Of the five Platonic solids, three are based on the crystal

group, the cube, tetrahedron and octahedron. The remaining two, the icosahedron and octahedron are the quasicrystal regular solids. That is, they cannot tile space alone or in combination with other Platonic solids. Virtually all quasicrystals discovered physically have the symmetry of the dodecahedron and icosahedron called *icosahedral symmetry*. As we go to 4D, we have four crystal symmetry polytopes, the 4D tetrahedron, 4D cube, 4D octahedron and a crystal based polytope called the 24-cell. We also have two quasicrystalline polytopes, the 4D icosahedron, called the 600-cell, and the 4D dodecahedron, called the 120-cell.

And then quasicrystalline symmetry ends. It never appears again in any dimension after 4D. Specifically, example, in every dimension higher, the only regular polytope are the hyper-tetrahedron (n-simplex), hyper-cube and hyper-octahedron.

Some have wondered why 3D and 4D appear especially related to our physical universe. If reality is based on quasicrystalline code, then this would perhaps be the reason.

Principle of Efficient Language

The *principle of efficient language* is the guiding law or behavior of the universe in the *emergence theory* framework. The old ontology of randomness and smooth spacetime is replaced by a code-based ontology, where symbolic information and meaning become the new first principles basis of our mathematical universe. As discussed above in the Symbolic Power of Fibonacci Chain Networks section in this Part IV of the paper, meaning comes in two fundamental categories: (1) ultra-low subjectivity physical meaning, which is purely geometric and (2) ultra-high subjectivity or virtually transcendent meaning, which includes things such as the meaning of irony and the myriad layers of meaning imposed by an experimenter about, say, the notion of a particle being measured as going through one slit or the other in a double-slit experiment. Interestingly, it is impossible to imagine an instance of ultra-high subjective meaning being disconnected from the underlying geometric code at the Planck scale. For example, the experience of humor is always associated with countless changes in particle position and alterations to the quantum,

gravitational and electromagnetic fields associated with that event. All forms of meaning are ultimately composed of actions of the quantum *viewing actions* that animate the code. The inherent non-local connectivity and distributed decision making actions of this neural-network like formalism allow various emergent patterns of intelligent choice and actualization of abstract meaning to be registered and considered within the degrees of freedom of the code. Choices will be made in such a manner as to create maximal associations and meanings but where meaning, in systems such as human beings, is highly subjective. Consider for example, how a joke can be told and one individual will react with massive levels of neural activity and associated meaning, while another person may barely comprehend it. The first person generates a much higher degree of correlated and physically meaningful actions when considered at the Planck scale level of the code operations. This feedback between the overall system (the universal emergent neural-network) and the person generating a larger amount of meaning from the joke plays a role in syntactically free choices of the code. We call these free choices the *hinge variable* steps in the code. On average, physical laws and actions are preserved because the physical meaning of the code (forces and physical laws) are the emergent and non-first principles manifestations of the underlying waveform language of the quasicrystalline quasiparticle formalism.

Phason Code

Phason quasiparticles have both a non-local wavelike quality and a local particle-like propagation aspect called a *supercell* in crystallographic parlance. As mentioned previously, there are three general ways matter can be organized: (1) Amorphous or gaseous materials have massive degrees of freedom and are there for not naturally codes. Geometric codes require a finite set of symbols, strict syntactical rules and minimal degrees of freedom. (2) Crystalline materials have no degrees of freedom unless there are local defects or phonon distortions. There are ultrafine scale vibrations allowed, but not organized code-based larger scale oscillations. (3) Quasicrystals are on maximally restrictive without being ultimately restricted like in the case of a crystal. For example, unlike a crystal, the assembly rules for a quasicrystal allow construction choices within the rules that are not forced. A crystal

allows only one possible type of relationship between atoms. For example, all vertex types in a cubic lattice are identical. In an amorphous or gaseous material, atoms can have a virtually infinite number of relational objects or vertex types. In a quasicrystal, as with any language, there is a rather small set of allowed combinations. For example, the Penrose tiling, which is found in nature, atoms form seven different allowed vertex geometries and the construction rules allow a very minimal level of freedom within the construction syntax.

Empires and Phason Flips

Because all quasicrystals are networks of 1D quasicrystals, understanding a phason flip and empires should start with how a quasicrystal is made via the *cut+projection* method. An irrational projection of a *cut* or slice of any crystal to a lower dimension produces a quasicrystal.

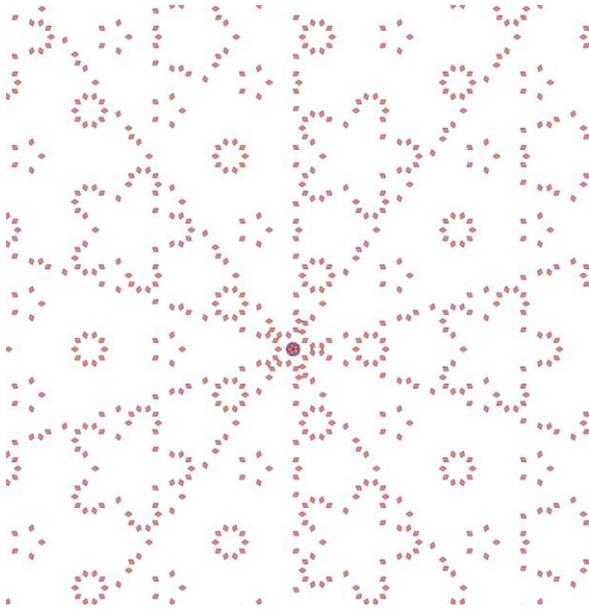
A rectangular cut window rotated with an irrational angle to the 2D crystalline pointset. One projects the points captured in the *cut window* to the 1D projection space to generate our 1D quasicrystal. In the second image, we translate the *cut window*, which projects a different set of points to the 1D space. When the *cut window* moves to a new coordinate, points instant jump in or out of possible positions in the 1D space that we call the *possibility space*. This instant change from one coordinate to the other in the *possibility space* is called a *phason flip*.

When one point is captured in the *cut window*, there are an infinite number of other points along the length of the *cut window* (if considering an ideal infinite point space) that are also captured in the *cut window* at its new coordinate. This creates an infinite number of *phason flips* in the 1D possibility space. An arbitrarily large but non-infinite quasicrystal can be built according to assembly rules [use German guy's evidence] instead of the *cut+projection* method. In this case, a user of the assembly language must choose a single *phason flip*, which is simply the designation of a point from the *possibility space* to be “on” or “off”. Notice from the above diagram that, when the *cut window*, changed location, some points in the 2D space (1) remained in the window, (2) some departed from the window and (3) some entered the window. When a

quasicrystal code user chooses a point to be “on” from the possibility *point space*, it causes a certain group of other points in the possibility space to also be turned on and other points to say on. These two sets of points are called the *empire* of the selected “on” point.

A key concept is conservation. The number of points captured in the *cut window* is conserved. As points enter the window, an equal number of points exit. A second key concept is non-locality, the *empire* of forced points determined to be “on” or “off” by a single point selection of a code user is very large. A third key idea is discrete and instant coordinate change. When the points are a model for particles, an ontology of instant coordinate change in the “physical” projection space is recognized, much like the notion of virtual particles in the *Dirac sea*, where particles are conserved such that when one is annihilated, another instantly appears.

Quasicrystals in dimensions higher than 1D are more complex because they are networks of 1D quasicrystals. So a *phason flip* and *empire* of a single 1D quasicrystal will have a massive empire that influences every other 1D quasicrystal in the network. Below is an image from Laura Effinger-Dean's thesis, which shows the *empire* of one of the *vertex types* of the Penrose tiling. We can see that the density of the *empire* drops with distance from the vertex being designated as “on” at the center. One can think of the possibility space as an aperiodic point space where any point can be selected to be one of the allowed *vertex types*. In the Penrose tiling, there are seven different vertex geometries. As mentioned, once one *vertex type* is selected for that vertex on the *possibility space*, it forces other vertices in the space to be “on” or “off” – the *empire*. A key point for physics modeling, where forces drop with distance, is that some *empires* drop in density with distance. We will connect this with the idea of *empire waves* and the *free lunch* principle shortly.



Another key idea for physics using this formalism would be that the *minimum quantum of action* notion of quantum mechanics would be replaced by the action of a point being registered as “on” or “off”.

Phason quasiparticle behavior in any quasicrystal has two distinct sets of construction rules:

1. Quasicrystal Assembly Rules – These construction rules govern how a single frozen state of selections on the *possibility space* can exist. The rules are defined by the angle, size and shape of the *cut window* in the higher dimensional lattice.
2. Ordering Rules for Two or More Quasicrystals – These rules govern the creation of dynamical patterns generated by ordering two or more different selection states on the *possibility space* into a stepwise frame-based animation. The rules are defined by the way that a *cut window* can translate or rotate through the higher dimensional lattice and whether combinations of those actions is discrete or continuous.

Empire Waves

Just as the 2D Penrose tiling quasicrystal has *empires* that are circular, with radial lines of higher density tiles evenly distributed from the *empire* center point, a 3D quasicrystal has *empires* with radial lines of higher tile

density penetrating evenly distributed points on a sphere. As explained in the section *Relativity - Electron clock*, a massive particle in our framework is composed of a vertex type (a *supercell* of twenty 3-simplexes) that dynamically animates over many coordinate changes or frames to forms (1) a toroidal knot cycle internally in the QSN and (2) a propagation pattern through the QSN that changes coordinate along a stepwise helical path. The interaction of these two forms of stepwise internal toroidal cycling and forward propagating helical cycling generates a richly complex dynamical pattern of empire waves – waves which extend to the end of the universal space of the QSN but drop in density with distance. These waves are the geometric first principles key to modeling forces in this framework as will be discussed next. First, however, it is helpful to explain that quantum mechanics does not require the assumption of Bohr’s conjecture known as *complementarity* – the core of the Copenhagen interpretation of quantum mechanics. This is the view that a fundamental particle, such as an electron, is either a wave or particle but never both. Neither experiment nor the mathematical machinery of quantum mechanics compel this interpretation. de Broglie-Bohm theory states that an electron, for example, is always a wave and particle at the same time and that the wave aspects guides the particle coordinate, like a pilot wave. The *cost* of this absolutely rigorous but less popular interpretation is the requirement of the assumption of inherent non-locality in nature. Empire waves are non-local according to the non-enigmatic geometric first principles of projective geometry.

Free Lunch Principle - Forces

With this general overview of *empire waves* established, it is now possible to understand how forces can be modeled via geometric first principles. Let us begin with the analogy of the game Scrabble, where you gain points by making multiple words diagonally or horizontally using letters from one or more other words already on the game board. When you do, this you get “free lunch” by earning points for each word your letter(s) played a position in. The Scrabble board is analogous to the QSN possibility space. And the 26 Roman letters are like the finite set of geometric relationships or *vertex types* in a quasicrystal – the geometric-symbols of the language. The rules and

freedom of English are like the rules and freedom of the phason code in a quasicrystal language. In Scrabble, the commodity that is to be conserved and used efficiently is the number of turns each player gets. Each turn needs to generate as much meaning as possible. In *emergence theory* physics, the same principle applies. There are a certain number of quasicrystal frames or “turns” that a system of, say, two particles can be expressed in over some portion of a dynamical sequence. Let us consider that it takes 10 frames to model a cycle of electron internal clock action or some total length of discrete transitions through the space. The patterns of this object in the QSN always need some integer ratio of the given number of total frames used for internal clock cycle steps versus helical propagation steps.

The physical pattern is expressed as the trinary selections of 3-simplexes in the QSN: on-right, on-left or off. And just as the words “cat” and “rat” can share an “a” for greater efficiency and synergy, the system of two such patterns moving through the QSN allow us to save steps. We can model *free lunch* in this geometric code thanks to the *empires*. When the first propagating electron is moving near the second electron, the two begin to benefit from one another’s *empire waves*. In the simplest example, consider that it would ordinarily take two remotely separated electrons 10 frames each to express a certain amount of clock cycling and propagation. However, the closer they are to one another, the more *free lunch* they will enjoy. The system saves frames when a selected tetrahedron from one particle’s empire is in the necessary right or left “on” state to that matches the state necessary to fill a position in the geometric pattern of a second electron, thereby saving a frame in the way that we saved an “a” in our Scrabble game example.

The result is that the particles require fewer phason flips or frames of trinary selections on the universal QSN to express their clock cycles and their given number of propagation steps along some direction. The physical meaning of this is that they have advanced a further distance than they would have otherwise with 10+10, where no *free lunch* is enjoyed. And because the density of *free lunch* opportunities increases with approaching distance, the two particles will accelerate toward one another as their separation decreases.

The *empire wave* around a massive particle in this framework is distinctly chiral and behaves according to the *right hand rule*, where the direction of propagation determines the direction of the chiral *free-lunch empire wave* system around it. A “train” of these objects, such as electrons in the QSN, will fall in-line behind one another and form a current because that positioning ensures the maximum amount of *free lunch*. By all moving along the same helical path, a group empire wave system, in the form of the chiral magnetic field, emerges around them. However, most electrons model are in either groups of free electrons or are in atomic systems that are arranged with many different orientations, such that the emergence of a chiral magnetic field does not occur. In other words, picture our model of the electron approaching Earth. As it accelerates closer, the probability of finding *free lunch* frame savings increases. Again, the *empire wave* field of every massive fundamental particle on Earth has no general similarity in their various orientations or directions of propagation. And they are not strongly correlated. Accordingly, around Earth, there is an enormous superposition of *empire waves* from every massive particle. One can say that it is a noisy quantum field of *empire waves* on the dynamical QSN. There is a high degree of non-coherence, as compared to a current of electrons, where there is are coherent group patterns in the *empire waves* – like combed flowing hair as opposed to tangled hair. Nonetheless, there will still be some opportunities for *free lunch* around the tangled array of *empire waves* surrounding large groups of massive particles for any approaching electron from outer space to enjoy as it nears Earth. But it will be exponentially less than the *free lunch* around the current of electrons. Gravity would logically be orders of magnitude weaker than electromagnetic forces. And it will be distinctly non-chiral, due to the fact that the average chirality is null, with an approximately equal quantity of right and left handed empire waves states on the QSN around Earth (other than the Earth’s magnetic field).

A Non-arbitrary Length Metric

The nearest neighbor lengths between points in the QSN are the Dirichlet integers 1 and $\frac{1}{\phi}$. So if our framework is generally correct, it would more deeply explain why black hole physics corresponds to the

golden ratio and why quantum mechanics does in the form of the φ^{-5} entanglement probability discovered by Lucien Hardy (65). Accordingly, a new length system based on golden ratio values would simplify many equations in physics. For example, the three most fundamental constants are the speed of light, c , the gravitational constant, G , and Planck's constant, h . The only number that unifies all three is a length called the Planck length, l , which happens to be about 99.9% of the golden ratio in the metric system.

$$l = \sqrt{\frac{\hbar G}{c^3}}$$

If spacetime had substructure built on our Planck length scale QSN, planetary systems might evolve overtime to energetically favorable cyclical and length ratios that approximate simple golden ratio fractions. And if we based our measuring system on a physical valued tied to a planet, it would be less arbitrary than, for example, the yard, which was based on the distance from King Henry I of England's nose to thumb distance.

Indeed, the metric system is less arbitrary because it is based on $\frac{1}{4}$ the circumference of Earth, where the distance from the Equator to the North pole is 10,002 kilometers, making the metric system unit value of 1, a full 99.98% of that distance (disregarding where the decimal is). When the system was established, they could not achieve the full accuracy of measuring this distance on Earth. So today the metric system unit is almost that distance. It is not well known, but the metric system deeply relates to approximations of golden ratio values. The Earth and Moon system is approximately a quarter of the age of the universe. So it has had a long time to self-organize into optimal ratios that approximate the golden ratio. To an accuracy of 99.96%, the dimensionless ratios are:

$$\left[\frac{\text{radius of Earth}}{\text{radius of Earth}}\right]^2 + \left[\frac{\text{radius of Moon} + \text{radius of Earth}}{\text{radius of Earth}}\right]^2 = \varphi^2$$

$$\text{Or } \left[\frac{\text{radius of Moon}}{\text{radius of Earth}}\right] = \sqrt{\varphi} - 1$$

In other words, this is a double coincidence. It is not just that the sum of the Earth and Moon diameters in the metric system are almost exactly the golden ratio 1.618..., but the breakdown of the two diameters that sum to that value is $\varphi - \sqrt{\varphi}$ for the Moon and $\sqrt{\varphi}$ for Earth.

The master dimensionless ratio of fundamental physics is the fine structure constant, a . Interestingly, it is also closely approximated with golden ratio expressions as:

$$a = \varphi^2/2\pi, \varphi^2/360$$

[to an accuracy of about 99.7%]

In our paper, *Emergence Theory and Astronomy*, myself and astrophysicist coauthor, Jesse Witbrod, layout a good deal of additional empirical golden ratio based physical data and consider the statistical probability of coincidence as the only explanation.

A Non-Arbitrary "Time" Metric As Ordered Quasicrystal Frames

Much of the data we present in that paper includes time based or planet and moon cycle "coincidences" that seem to match far too closely to the golden ratio to be explained away by anything other than the presumption of some unknown substructure of spacetime in a new quantum gravity framework.

By combining both time and length based values, the critical reader can perhaps be interested in the following impressive number.

The gravitational constant, G , ties time and length based values together as:

$$G = \chi c^2/4\pi$$

And $h\chi = 1.0000026$ of the golden ratio as $0.6180382(10^{-53})$ cubic meters per second [note $1/\varphi = 0.61803...$ and $\varphi = 1.61803...$ are the same ratio]. This deviation at the millionth place after the decimal is remarkable.

Now, having put forth an argument why it is plausible that spacetime can have a golden ratio based

substructure as a natural result of the projection of E_8 to a lower dimensional quasiperiodic point space, we can speculate on the idea that the metric system is deeply related to ϕ and consider the idea of a first principles analytical expression of the constants c , G , h and a . But clearly, there is a problem. The first three constants are dimensional and tie into the speed of light. And the speed of light is based on a length metric and a time metric. The length metric is being proposed as non-arbitrary, according to this speculative argument related to our projective approach to E_8 unification physics.

However, the speed of light playing into these equations corresponding appears at first to be based on an arbitrary metric for time, the second. The golden ratio based Earth Moon and Sun system. The QSN is based on the numbers 2, 3 and the golden ratio because the 3-simplex building blocks are regular or non-distorted. And the golden ratio is deeply related to 5 geometrically in the form of the pentagon and to 5 algebraically as $\frac{1}{2}$ of $\sqrt{5} + 1$. From the analytical expressions of the 3-simplex volume to its length values, such as height and centroid to vertex distances, it is fundamentally built of the numbers 2 and 3 and their square roots. So the QSN is deeply related to 2, 3 and 5. Incidentally, these are the symmetries that define anything with icosahedral symmetry. And nearly every quasicrystal found in nature (over 300) possesses icosahedral symmetry. It is interesting, then to note, therefore, that the constant c (in the metric system) is 99.93% of the number 3, disregarding where the decimal is placed. And the distance of the Earth to the Sun is 99.73% of $3/2$.

The number of (presumably) ϕ based meters traveled by a photon in vacuum in one second is a close approximation of $3/2$. Assuming hypothetically, that E_8 quasicrystalline physics is a good approach, why is this the case if the second is arbitrary?

The second is not arbitrary, of course. It is based on a cycle of the fundamental Earth clock system, which itself is fundamentally based on ϕ , as argued above. It is based on the clock cycles of the Earth rotating once on its axis, which is gravitationally and electromagnetically tied to the Earth, Moon, Sun system as a whole. The number, of course, is 86,400

seconds in one of these non-arbitrary physical cycles of the Earth clock. That is, 60 seconds x 60 minutes x 24 hours. Remarkably, the modern precise average Earth day is 86,400.002 seconds. So the old number is unexpectedly close to the accurate measurement. Again, we disregard where the decimal place is in the context of thinking about the fundamental aspect of a number – its factorization. Accordingly, 86,400 becomes $864 = 2^5 3^3$, a number deeply related to 2, 3 and 5.

Have we missed anything obvious? Yes, the Earth distorts along the equator. So if we adjust for the meter to assume a non-rotating Earth with no distortion, we can see if our number gets closer or further from the golden ratio being the Planck length. Realizing that the pole through pole diameter of Earth is 12,713 km, and simplifying the value by moving the decimal to 1.2713, we can calculate that a $\frac{1}{4}$ of a circle intersecting a non-distorted sphere of this diameter is .9984766. This then is normalized to 1. Note that this approach is not based on a metric. It is based on the ratio of the Moon to Earth, where we get the dimensionless value. And here we are not using the ideal Phi values mentioned. We are using the actual values in their ratio. So this gives the dimensionless ratio value of .9984766 in the manner just described. We can then normalize this to a standard unit of 1. Again the justification is the conjecture that the substructure of space is based on a dimensionless ratio of one part being 1 and the other part being $1/\phi$. Now, what this means is that the Planck length now changes slightly from the current value of 1.616199, which is based on the meter that is measured from a distorted Equator to the normalized value based on the new dimensionless ratio based length and based on the actual measurement of the Earth's pole through pole diameter (not plugging the golden ratio approximation of that diameter). We get a logically adjusted Planck length of 1.6183412... or 1.0002 of the golden ratio.

Mass in the Quasicrystalline Spin Network

We will now combine the following ideas in order to understand mass in the *emergence theory* framework:

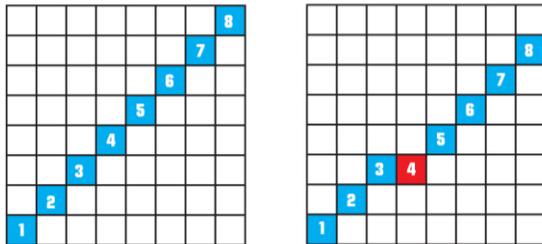
1. Free lunch and empire waves

2. Massive particle clock time to propagation inverse relationship
3. Principle of efficient language

Obviously, our vision of a geometric first principles theory of everything, as explained thus far, is a program of reducing everything to length. Our formalism is Clifford rotor operations on a spin network made of the two Dirichlet integer values 1 and $1/\phi$.

Mass is the degree of resistance to a change in direction or acceleration of a massive particle. If space and time are discretized, where space divided into positions like on a checker board and time is divided into turns of the players, where a piece can only move to a connected square, an intuitive understanding of mass emerges.

In this diagram, we see that putting a particle in motion along some direction spacetime as the checkerboard achieves an efficient diagonal progression across the board. However, a change in direction



First principles explanation of c, h and G
 An explanation of the regularization scheme and how the 15.522° thing plays in and the benefits of it and the two ways of expressing geometric frustration.

GENERATION OF THE QUASICRYSTALLINE SPIN NETWORK

Fang Method – In Method I, Fang Fang initially constructed the QSN by modifying the *icosagrid* with Fibonacci chain spacing to make it a quasicrystal. In doing this, the alternative Method II was discovered. Packings of tetrahedra in the form of the FCC lattice are Fibonacci chain spaced. Then five copies are rotated from one another by the 15.522° angle.

Method I – This approach is inspired by the *pentagrid* method of constructing the Penrose tiling. A 3D analogue is the *icosagrid* construction method for icosahedrally symmetric quasicrystals. 10 sets of equidistant planes parallel to the faces of an icosahedron are established with periodically repeating parallel planes in each set that, together, form the icosahedrally symmetric *icosagrid*. The intersecting planes segment the 3-space into an infinite number of 3D cells sizes. The *icosagrid* is not a quasicrystal due to the arbitrary closeness of its edge intersections and the resulting infinite number of prototile sizes. We converted it into an icosahedral quasicrystal by changing the equal spacing between parallel planes to have a long and short spacing, L and S with $L/S =$ golden ratio and the order of the spacing follow the Fibonacci sequence. Therefore we call this kind of spacing the Fibonacci spacing. This Quasicrystal turned out to be a 3D network of Fibonacci chains and we would like to name it Quasicrystalline Spin-Network.

Method II: Golden Composition of the Fibonacci Tetra-grid

Similar to the *icosa-grid*, a *tetra-grid* is made of 4 sets of equidistant planes that are parallel to the faces of a tetrahedron. Apply the Fibonacci spacing to this structure will also give us a quasicrystal with tetrahedral symmetry (Fibonacci *tetra-grid*) – again we focus mostly on the regular tetrahedral cells. In order to obtain icosahedral symmetry, we need to implant the 5-fold symmetry. We applied a Golden Composition process to this Fibonacci *tetra-grid* and achieved the same QSN structure.

The Golden Composition is described as follows:

1. Start from a point in a Fibonacci *tetra-grid* and identify the 8 tetrahedral cells sharing this point with 4 in one orientation and the rest 4 in another orientation
2. Pick the 4 tetrahedral cells of the same orientation and duplicate another 4 copies.

Put two copies together so that they share their center point, the adjacent tetrahedral faces are parallel, touching each other and with a relative rotation angle of $\text{ArcCos}([3\phi - 1]/4)^\circ$, the *golden rotation*. Repeat the process three more times to add the other three copies to this structure. A twisted 20-tetrahedra cluster, 20G, is formed in the end. Now expand the Fibonacci *tetra-*

grid associated with each of the 4-tetrahedron set by turning on the tetrahedra of the same orientation as the four, an icosagrid of one chirality is achieved. Similarly, if the tetrahedral cell of the other orientation are turned on, an icosagrid of the opposite chirality will be achieved. In either case, there is a 20G at the center of the structure.

Clifford Rotor Induction Method

In Sen et. al. [publication pending], an inductive framework has been established to link higher dimensional geometry from the basic units of an icosahedron using spinors of geometric algebra and a sequence of transformations of Cartan sub-algebra. Spinors are linear combination of a scalar and bivector components defined as follows:

$$s = \kappa_D + \alpha_D \mathbf{e}_{12} + \beta_D \mathbf{e}_{23} + \gamma_D \mathbf{e}_{31}$$

The subscript D denotes that the spinorial coefficients live in a Dirichlet coordinate system, i.e. $\alpha_D := \alpha_1 + \phi\alpha_2$, where ϕ is the *golden ratio*. This approach presents, for the first time to our knowledge, a direct inductive and Dirichlet quantized link between a three dimensional quasicrystal to higher dimensional Lie algebras and lattices that are potential candidates of unification models in physics. Such an inductive model bears the imprints of an *emergence* principle where all complex higher dimensional physics can be thought to emerge from a three dimensional quasicrystalline base.

Dirichlet Integer Induction Method

The need for quasicrystalline coordinates brought us naturally to consider a class of number which are more rich than the rational integers (useful for crystals), but more constraint that the real numbers, the quadratic integers. From this class, the five-fold symmetry of our quasicrystal guides our choice to the ring living in the quadratic field associated to 5, which is sometimes noted $\mathbb{Z}[\phi]$, the quadratic ring of “Dirichlet integers” referencing to their use in Dirichlet’s thesis and following works, or in short \mathbb{D} .

Then we use a digital space \mathbb{D}^3 , to host triplets of Dirichlets integers, and a digital spacetime \mathbb{D}^4 , to host quadruplets of Dirichlets integers. \mathbb{D}^4 could have a quaternion structure, and written $\mathbb{H} \otimes \mathbb{D}$. Where \mathbb{D}^3 the space part, is the imaginary part. Furthermore the structure can be complexified to biquaternion, and also put in bijection with octonion and sedenion. Each point can also be seen as a 4 by 4 matrix of integers, a digital tetrad, $M^4(\mathbb{Z})$, or as $\mathbb{H} \otimes \mathbb{H} \otimes \mathbb{Z}$.

The 16 numbers are integers.

$$1 \quad i \quad j \quad k \cdot \begin{matrix} a_w & b_w & c_w & d_w & 1 \\ a_x & b_x & c_x & d_x & \phi \\ a_y & b_y & c_y & d_y & I \\ a_z & b_z & c_z & d_z & I\phi \end{matrix}$$

There is one line per dimension, and the first dimension, indexed by w is hidden in the space construction. The a and b are combined to make the real part of the Dirichlet complex.

The generators satisfies: $ijk = i^2 = j^2 = k^2 = I^2 = \phi - \phi^2 = -I$. In a first approach, the imaginary part will be set to 0 (so all c and d are null). A point in the realized space will just show three coordinates:

$$\begin{matrix} a_x + \phi b_x \\ a_y + \phi b_y \\ a_z + \phi b_z \end{matrix}$$

...where $\phi = \frac{1+\sqrt{5}}{2}$, and is equivalent to the non-golden part made of the a , and the golden part made of the b .

In a Euclidian spacetime, a_w and b_w can correspond to time, while it is c_w and d_w in a Lorentzian spacetime, and all four are used in a Kaluza-Klein model.

A set of eight integers, (the a and b , or the c and d), can encode a position in an E8 lattice (with a doubling convention).

Let us focus on how this numbers emerge. Our model is from first principle built from regular tetrahedral in an Euclidian 3D space, because the simplex is the simplest geometric symbol, and the space is observed as tridimensional.

We asked the question: which sets of vertices in \mathbb{D}^3 can hold regular tetrahedral of the same size having one vertice in the center (0,0,0). The equation is the equation of the sphere, written in \mathbb{D}^3 , which hold two equations by separating the golden and the non-golden parts.

Having built a first principle version of Dirichlet space where 20G emerge naturally (but as 4 copies). I have the intuition that E8 physics can also emerge naturally as encoded by the possible tetrahedron configurations. We will focus on rule emergence.

Some from physics, like the hadronic rule saying that the combined color of three quarks in a neutron or photon is neutral, also known as the SU(3) symmetry, quantum chromodynamics. Some from information

theory and mathematics. Some from matching with quasicrystal study when importing CQC simulation with dynamic window and observing phason occurrence, to deduce phason rules.

The digital space is compatible with different versions of the FIG and ready for integrating all possible regularization of 4DQC based on 600-cell, according to the team fixed rule: regular tetrahedral

A rigorous analytic approach using equations (Dirichlet sphere, giving two equations)

A maximal possibility space biggest than FIG but smaller than Dirichlet space

First rules: checkerboard rule, golden rule

Vertex figure: new polyhedron with 108 vertices and 86 faces

Tetrahedron centers figure: new polyhedron with 32 vertices (but not the ID)

Tetrahedron possibility states: 24 as *trits* (on-right, off-right and off), eight as 2*bits (off or three orientations)

72 possible tetrahedron around a vertex

4 possible 20-Groups around a vertex (2 by chirality * 2 by rotation)

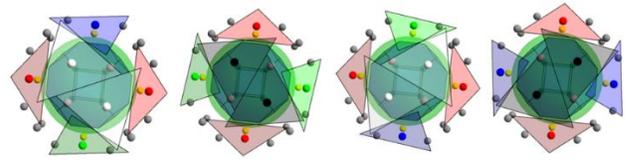
Mathematical definition of the point space as binary 6-hypermatrix. For example rule 1: $a_{ijklmn}=1$ implies $i+j+k$ is even and $l+m+n$ is even rule 2: $a_{ijklmn}=1$ implies that there exist 3 other sets of indices having also $a_{ijklmn}=1$ where $(I-i+\varphi(L-l))^2+(J-j+\varphi(M-m))^2+(K-k+\varphi(N-n))^2=8$

Rule 1, the “checker board rule” is one of the theorems we have to prove in my paper (as we discussed recently)

Rule 2, the ‘golden rule’ stipulate that our vertices are not isolated but belongs to regular tetrahedra, when the 6D Dirichlet integer space is projected to the 3D real space

A binary 6-hypermatrix is isomorph to a finite set of 6d integer coordinates points

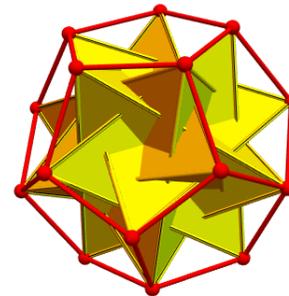
I expect to show that this binary 6-hypermatrix is the visible face of a binary 8-hypermatrix ‘iceberg’ encoding also the quantum nature of the particles (and its 8 quantum numbers in E8 model); Or a 16-hypermatrix if complexified.



Projection and Graph Diagram Method

To illustrate the correspondence of the 20G twist to the E_8 lattice, I developed the following method:

1. Project E_8 to 4D to generate the the Elser-Sloan quasicrystal. It is made entirely of 600-cells. Alternatively, we may project one of the 240 vertex root vector polytopes of E_8 to 4D to generate two 600-cells scaled by the golden ratio.
2. Select 20 tetrahedra sharing a common vertex in a 600-cell and project the cluster to 3D such that the outer 12 points form the vertices of a regular icosahedron.
3. Induce its dual, the dodecahedron, which has 30 points.
4. Use the 30 points to create a graph diagram by connecting points separated by a distance of $\varphi(\sqrt{2})$ times the dodecahedral edge length. This creates a 3D graph diagram equal to two superimposed tetrahedron 5-compounds, one right and one left handed.



In the above image, the right chirality 5-compound is shown. The Cartesian coordinates of the 30 vertices are the cyclic permutations of:

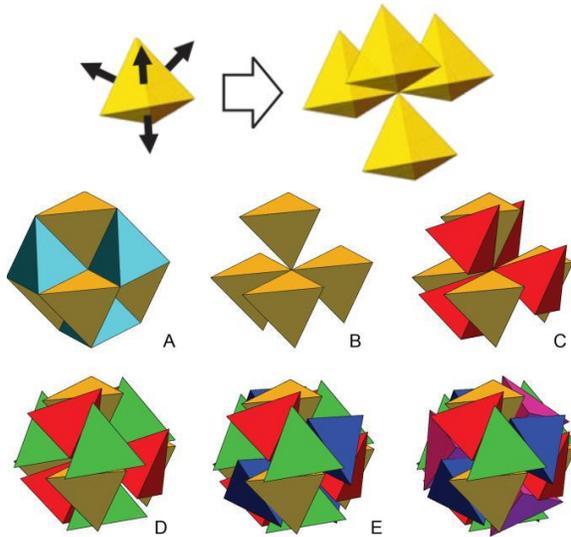
$$(\pm 1, \pm 1, \pm 1)$$

$$(0, \pm 1/\varphi, \pm \varphi)$$

$$(\pm 1/\phi, \pm \phi, 0)$$

$$(\pm \phi, 0, \pm 1/\phi)$$

5. Select either the right or left handed tetrahedron 5-compound. And from it, we select one tetrahedron and translate a copy of it away from the center of the cluster along one of its 3-fold axes of symmetry by a distance of $\sqrt{3}/8$ times its edge length – the distance necessary to translate one of its vertices to be coincident with the center of the tetrahedron 5-compound. We do the same copy-and-translate action along the other three of the selected tetrahedron's 3-fold axes of symmetry. This generates four new tetrahedra that share a common vertex with the centroid of the cluster. Their 12 outer vertices form the points of a cuboctahedron.



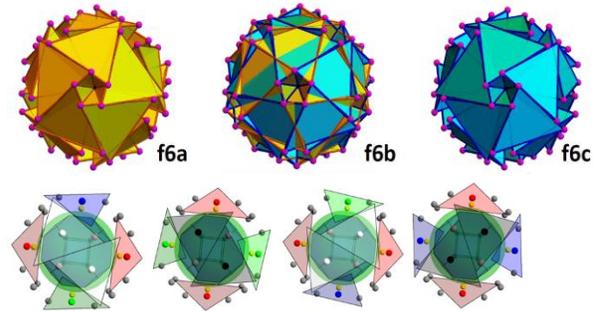
We repeat this 4-step process with the remaining four tetrahedra from the initial tetrahedron 5-compound. We then remove the original five tetrahedra as well as the dodecahedron. Thus far, this induction process generated 20 tetrahedra sharing a common vertex at their group center. It is the 20G twist with 60 outer vertices equal to a cuboctahedron 5-compound. It has Cartesian coordinates that are the cyclic permutations of:

$$(\pm 2, 0, \pm 2)$$

$$(\pm \phi, \pm \phi^{-1}, \pm (2\phi - 1))$$

$$(\pm 1, \pm \phi^{-2}, \pm \phi^2)$$

6. Finally, we repeat the entire process with the starting tetrahedron 5-compound of the opposite chirality. The right handed 20G twist [f6a] and left handed one [f6c] are superimposed in the QSN to form the basic building block set of possibilities from 60+1 points and 180 possible 3-simplex edges or connections [f6b].



Hyperdimensional Information Encoded in 3D

One of the principles of the *emergence theory* approach is simplicity. We question the physical realism of hyperdimensional spaces implied by models such as general relativity and string theory.

However, it is clear that the gauge symmetry unification of all particles and forces of the *standard model*, which is everything except gravity, are described by the 6-dimensional root vector polytope of the E_6 lattice. Full gauge symmetry unification with gravity seems to be possible with the 8-dimensional E_8 lattice, which embeds E_6 . This unification can be achieved with or without geometry by selecting either the pure algebraic Lie algebras or the geometric analogues of hyperdimensional crystals and geometric algebras. Even without considering the gauge theory implications of hyper geometry, general relativity alone relies on a 4D geometric structure.

Clearly fundamental physics implies a deep tie to hyperdimensional math. But we have never measured any geometric dimensions beyond 3D. So the Occam's razor approach is to see if we can derive all the hyperdimensional information from a purely 3D framework without having to adopt the ontological realism of, for example curled up spatial dimensions or the 4D spacetime of general relativity.

Because we can measure reality to be 3D and because it is simpler than hyper-spaces, we seek to model the implied math of hyperdimensional geometry while restricting ourselves to Euclidean 3-space.

The quintessential example of a lower dimensional object encoding the information of a higher dimensional object is irrational angle based projective geometry – quasicrystallography.

The vertex Cartesian coordinates are the cyclic permutations of:

$(\pm 1, \pm 1, \pm 3)$
 $(\pm \varphi^{-1}, \pm(-\varphi^{-2}), \pm 2\varphi)$
 $(\pm \varphi, \pm(-2\varphi^{-1}), \pm \varphi^2)$
 $(\pm \varphi^2, \pm(-\varphi^{-2}), \pm 2)$
 $(\pm(2\varphi^{-1}), \pm 1, \pm(2\varphi - 1))$

Emergence of a Self-actualized Code Operator

Frank Wilczek's challenged physicists to develop a conscious measurement operator that comports with the formalism of quantum mechanics (50). This is daunting for social reasons. Discussions of consciousness in academic circles of physicists is generally scorned, with few exceptions. And many physics journals reject such notions under the unwritten premise that philosophy and physics should not be combined.

However, blunt logical deduction, free of social fears, points to the idea that consciousness is a fundamental element, as though it is the substrate of reality.

J.B.S. Haldane (93) said:

We do not find obvious evidence of life or mind in so-called inert matter...; but if the scientific point of view is correct, we shall ultimately find them, at least in rudimentary form, all through the universe

Erwin Schrödinger (94) said:

For consciousness is absolutely fundamental.

Andrei Linde (95), co-pioneer of inflationary big bang theory, said:

Will it not turn out, with the further development of science, that the study of the universe and the study of consciousness will be inseparably linked, and that ultimate progress in the one will be impossible without progress in the other?

David Bohm (96) said:

The laws of physics leave a place for mind in the description of every molecule... In other words, mind is already inherent in every electron, and the processes of human consciousness differ only in degree and not in kind.

Freeman Dyson (96) said:

That which we experience as mind... will in a natural way ultimately reach the level of the wave-function and of the 'dance' of the particles. There is no unbridgeable gap or barrier between any of these levels... It is implied that, in some sense, a rudimentary consciousness is present even at the level of particle physics.

Werner Heisenberg (97) said:

Was [is] it utterly absurd to seek behind the ordering structures of this world a consciousness whose "intentions" were these very structures?

The growing credibility of the digital physics argument still leaves one with the sense of audacious improbability. These scientists claim that the universe is a simulation in the quantum computer of an advanced being or society. Although they could be correct, this has a similar level of outlandishness as the idea that a creator *God* from outside the universe is the source of everything. Of course, this is a popular religious view. But the idea that something from outside the universe created the universe implies a new definition of the term *universe*. That term is supposed to mean *everything*. The idea of a self-actualized universe may be more sensible.

The mounting evidence that the universe is made of information and is being computed includes the

aforementioned mathematical proof of the *Maldencena conjecture* and the discovery of error correction codes. There are many other pieces of evidence that add to the argument. But for those new to the thought process, here is a simple way to deduce that something like a computer or mind is needed: Everything we know about physics, including classic physics, indicates that reality or energy is information. And information cannot exist without something to actualize it. It is abstract and relates deeply to a mind-like entity, whether that be a biological neural network or an artificial intelligence.

However, there is a more plausible explanation than the digital physics computer simulation hypothesis. In his submission to the FQXi Essay contest, mathematical physicist, Ray Aschheim (7), a scientist here at Quantum Gravity Research, said:

Can reality emerge from abstraction, from only information? Can this information be self-emergent? Can a structure be both the software and the hardware? Can it be ultimately simple, just equivalent to a set? Can symmetry spontaneously appear from pure mathematical consideration, from the most symmetric concept, a Platonic "sixth element"? Would this symmetry be just structuring all the particles we know? Can all this be represented? Can standard physics be computed from this model? Eight questions: eight times yes.

The notion of a self-emergent computational but non-deterministic neural-network universe is more plausible than the idea of a simulation creator from outside the universe. In fact, the emergence of freewill and consciousness need not be a speculation. It is proven to exist, at least in humans. So it is one of the interesting behaviors of the universe locally in the region of our physical bodies. In my paper, *A New Approach to the Hard Problem of Consciousness: A Quasicrystalline Language of "Primitive Units of Consciousness" in Quantized Spacetime* (98), I discuss in detail the plausibility of a self-emergent mind-like universe. The first question is to consider whether or not physics imposes a limit on self-organizing evolution of consciousness. In other words, are humans the limit or can intelligence tend toward infinity? From what we know about classic and quantum physics, there is no limit. It can tend toward infinite awareness and

intelligence. The next question is, "What percentage of the energy in the universe can self-organize into conscious systems and networks of conscious systems?" Of course, the answer is the same as the first question. Physics imposes no upper limit. So the answer is that, in principle, 100% of the energy of the universe can self-organize into a conscious network of conscious sub-systems systems. The final consideration in the deduction relates to the axiom:

Given enough time, whatever can happen will happen.

By this axiom, somewhere ahead of us in spacetime, 100% of the universe has self-organized into a conscious system. It certainly need not be anthropomorphized. We can leave the detail of what this entity would be like out of the deduction. For example, there is no reason to presume that it cannot exist trans-temporally and have an extremely different quality than what we conceptualize as consciousness. The next step of deduction is to question whether or not trans-temporal feedback loop are disallowed by current physics paradigms.

Stephen Hawking of Cambridge and Thomas Hertog of the European Laboratory for Particle Physics at CERN say that the future loops back to create the past (99). The *delayed choice quantum eraser* experiment also indicates that the future loops back to create the past. And Daryl Bem of Cornell has published several experimental results demonstrating retro-causality (57). In 2014 Brierley et al. (100), demonstrated quantum entanglement of particles across time. In fact there is an *oldwives tale* that general relativity prohibits trans-temporal feedback loops. This is not true. General relativity simply states that communication between events cannot occur via photon mediation. In fact, general relativity predicts wormholes through time and space. The inherent non-locality of quantum reality does not require signals for things to be connected; any more than rotating a penny while looking at the heads side requires time to transmit the torque to the other side of the penny. It is a simultaneous or null-speed correlation. The truth is that, until we have a predictive first principles theory that unifies general relativity and quantum mechanics, one cannot aggressively invoke interpretations of either of these two *place-holder* theories to say with confidence what can and cannot occur. Both will turn out to be flawed or incomplete in

certain ways when a full theory of everything is discovered.

So for now, let us develop the most conservative argument as follows:

1. Like the exponential explosion of any doubling algorithm, high level forms of consciousness and networked consciousnesses will envelope the universe. There are no hypotheses that can reasonably challenge this idea. We have hard evidence that consciousness emerges because our minds are sharing the words of this sentence. The idea of consciousness exponentially spreading throughout the universe is plausible due to the extraordinary behavior of doubling algorithms. For example, if we doubled a penny as fast as we can hit the “x2” button on an iPhone calculator in 30 seconds, we would have more pennies than all the atoms in the entire universe. The reason we do not see doubling algorithms in nature go more than a few iterations is because resources halt the doubling algorithm very early.
2. The question is whether or not a species with high consciousness and evolving consciousness can leave their biosphere and continue doubling and staying non-locally networked. Humans made it to another cosmological body in 1969, when we landed on the Moon. It is only a matter of time before technology and our built-in compulsion to explore takes us out into the universe, where resource limitation halting will not occur until all the energy of the universe is exhausted. Again, the challenge is not to argue why this will occur. That is established by the axiom “Given enough time, whatever can happen will happen”. The onus of logic falls on those who guess humans will destroy themselves or that society will collapse or and that all other potential species in the universe will have the same fate.
3. Now, if the universe is expanding faster than the speed of light, then exponentially expanding consciousness can never sequester all energy into a universal scale conscious neural network of quantum entangled conscious sub-systems. As mentioned, general relativity allows wormholes, and quantum mechanics is inherently non-local. So

until a predictive theory of everything is discovered, it is not clear whether or not non-local information exchange or teleportation can occur, where a consciousness can relocate trans-temporally or trans-spatially in instant-time (perhaps without atomic form) to influence matter and energy in distant regions of the universe. However, it is worthwhile to play the *what-if* game to see where the idea leads. If a new first principles quantum gravity theory inspired a technology that allowed consciousness to project into spacetime coordinates non-locally, where would we go first? *What if* you were given 100 free airline vouchers to fly anywhere in the world? Would you explore ballistically by first traveling 100 miles from your home, then 200 miles and so on until you explored the far reaches of the world? Or would you make a *favorites list* and bounce around arbitrarily depending on whether Beijing, Sydney or Rio makes it near the top of your wish list? If humans or any other intelligent life in the universe discovers non-local information exchange that a consciousness can exploit, we will bounce around the universe and plant consciousness in various parts of the cosmos. At first, the outposts transplanted consciousness will have the sparse pattern of a sponge – or neural network – throughout the whole cosmos. So the expansion rate of the universe would not be a problem for the deduction that, given enough time, high consciousness will eventually envelop all energy in the universe. The sparse sponge-like pattern of *outposts* of networked consciousness will fill-in as they approach maximum density at 100% of the energy of the universe.

4. What would this high consciousness be like? It is hard to say. But it would not be very much like us. We are related to snails and horses and dinosaurs. But we are not very much like them. However, the one thing we would share in common is that we would understand the first principles theory of everything that would be a prerequisite for the exploitation of non-local mental and physical technology. When a first principles theory of everything is discovered, it will not be replaced by something else. To say otherwise indicates a misunderstanding of what the term *first principles* means in this context. The Pythagorean Theorem

is based on first principles. It will not be replaced. We are not talking about a model of how the universe works. We are speaking of discovering the simulation code of geometric symbolism itself and interacting with it.

So we have told an audacious story, even though it may be logically inevitable. However, it should be noted that the *big bang* theory is audacious and probably true at the same time. The emergence of this very conversation, dear reader, and the human consciousness that it exists within is audacious. And so too is the notion of the universe being a simulation from a creator outside the universe. So of audaciousness is evil, then we are seeking the lesser of all evils. The deduction herein is in fact conservative. And yet it is audacious at the same time. It is not just plausible. It is inevitable.

The punchline of the deduction is this: Because this is an inevitable outcome, the simplest answer on how an information theoretic universe can exist and what its substrate is would be that it self-actualized. The entire system – reality – is a mind-like mathematical (geometric) neural network. Just as our now limited consciousness can hold within it the notion of a square, we can allow a self-organizing game or language of squares to emerge in our mind. A far greater neural network could hold within it the relatively simple geometry of E8 and the 4D and 3D quasicrystals we have discussed. Primitive quanta or measuring entities (*quantum viewers*) at the Planck scale substructure of the imagined *possibility space*, which are essentially vantage points of the universal emergent consciousness, would actualize geometric symbols by *observations* (projective transformations) within the quasicrystalline *possibility space*. Each observation of a Planck scale *quantum viewer* generates a projective

transformation equal to a rotation of the 3-simplex it is associated with. These primitive geometric binary choice state on the *possibility space* are part of a code that forms a neural network based on 3D simplex-integers in an E₈ derived quasicrystal. By geometric first principles, the code has a free-variable called the *phason flip*. And the universal consciousness operating the details of the code obeys the *principle of efficient language*, taking instructions from conscious sub-systems like us, who are engines of emergent meaning.

The universe would not exist if it weren't for intermediary emergent entities like us. It would also not exist if it weren't for the maximally simple golden ratio based quasicrystalline E₈ code that self-organizes quarks and electrons into 81 stable atoms and into countless compounds and planets and people and societies and overly-wordy sentences and on up through to the collective consciousness of the universe. And that primitive starting code and the simplex-integers and the *quantum viewer* operators needed to animate the whole thing would not exist without the collective emergent consciousness. Retrocausality allows the whole idea to be logically consistent, where the future creates the past and the past creates the future. The simple creates the complex and the complex creates the simple – a cosmic scale evolving feedback loop of co-creation. This framework is both explanatory and conservative. And it requires no magical moments that are unexplainable, like the moment of the *big bang* or a creator-*God*. It uses first principles logic where A co-creates B, which co-creates C, which co-creates A. Non-linear causality is mathematically and logically rigorous. The entire framework is based on two fundamental and inarguable behaviors of nature: (1) emergent complexity and (2) feedback loops.

Bibliography

1. *Colloquium: Physics of the Riemann hypothesis*. Schumayer, Dániel and Hutchinson, David A. W. s.l. : APS, 2011, Reviews of Modern Physics, Vol. 83, p. 307.
2. *Birds and frogs*. Dyson, Freeman. 2009, Notices of the AMS, Vol. 56, pp. 212-223.
3. Lapidus, Michel Laurent. *In Search of the Riemann Zeros: Strings, fractal membranes and noncommutative spacetimes*. s.l. : American Mathematical Soc., 2008.
4. *Unity of all elementary-particle forces*. Georgi, Howard and Glashow, Sheldon L. s.l. : APS, 1974, Physical Review Letters, Vol. 32, p. 438.

5. *A universal gauge theory model based on E6*. **Gursey, Feza, Ramond, Pierre and Sikivie, Pierre**. s.l. : Elsevier, 1976, Physics Letters B, Vol. 60, pp. 177-180.
6. *An exceptionally simple theory of everything*. **Lisi, A. Garrett**. 2007, arXiv preprint arXiv:0711.0770.
7. *Hacking reality code*. **Aschheim, R.** 2011, Preprint.
8. *Online Dictionary of CRYSTALLOGRAPHY*. [Online] http://reference.iucr.org/dictionary/Aperiodic_crystal.
9. *On the distribution of the roots of certain symmetric matrices*. **Wigner, Eugene P.** s.l. : JSTOR, 1958, Annals of Mathematics, pp. 325-327.
10. *Primes, quantum chaos and computers*. **Odlyzko, Andrew M.** 1990. Number Theory, Proceedings of a Symposium. pp. 35-46.
11. *The pair correlation of zeros of the zeta function*. **Montgomery, Hugh L.** 1973. Proc. Symp. Pure Math. Vol. 24, pp. 181-193.
12. *Random matrices: The universality phenomenon for Wigner ensembles*. **Tao, Terence and Vu, Van.** 2012, Modern aspects of random matrix theory, Vol. 72, pp. 121-172.
13. **Deza, Michel, Grishukhin, Viatcheslav and Shtogrin, Mikhail.** *Scale-Isometric Polytopal Graphs In Hypercubes And Cubic Lattices: Polytopes in Hypercubes and Zn*. s.l. : Imperial College Press, 2004. Remark 14.5.
14. **Hopcroft, John E.** *Introduction to automata theory, languages, and computation*. s.l. : Pearson Education India, 1979.
15. *An Icosahedral Quasicrystal as a Golden Modification of the Icosagrid and its Connection to the E8 Lattice*. **Fang, Fang and Irwin, Klee.** 2015, arXiv preprint arXiv:1511.07786.
16. *Statistical mechanics of cellular automata*. **Wolfram, Stephen.** s.l. : APS, 1983, Reviews of modern physics, Vol. 55, p. 601.
17. *Algebraic properties of cellular automata*. **Martin, Olivier, Odlyzko, Andrew M. and Wolfram, Stephen.** s.l. : Springer, 1984, Communications in mathematical physics, Vol. 93, pp. 219-258.
18. *A heuristics for graph drawing*. **Eades, Peter.** 1984, Congressus numerantium, Vol. 42, pp. 146-160.
19. Simplex. *Mathworld*. [Online] <http://mathworld.wolfram.com/Simplex.htmlref>.
20. **Gauss, Carl Friedrich and Schering, Ernst.** *Carl Friedrich Gauss Werke...* s.l. : Gedruckt in der Dieterichschen Universitäts-Buchdruckerei, 1874. Vol. 6.
21. *Note on Riemann's ζ -Function and Dirichlet's L-Functions*. **Ingham, A. E.** s.l. : Oxford University Press, 1930, Journal of the London Mathematical Society, Vol. 1, pp. 107-112.
22. *The Density of Primes*. **Fine, Benjamin and Rosenberger, Gerhard.** s.l. : Springer, 2007, Number Theory: An Introduction via the Distribution of Primes, pp. 133-196.
23. *The search for a Hamiltonian whose Energy Spectrum coincides with the Riemann Zeta Zeroes*. **Raymond Aschheim, Carlos Castro Perelman, Klee Irwin.** https://www.researchgate.net/publication/307857776_The_search_for_a_Hamiltonian_whose_Energy_Spectrum_coincides_with_the_Riemann_Zeta_Zeroes : Submitted for Publication, 2016.
24. *The Code Theoretic Axiom, The Third Ontology*. **Irwin, K.** s.l. : Submitted for publication, 2017.
25. *The microstructure of rapidly solidified Al6Mn*. **Shechtman, D. and Blech, I. A.** s.l. : Springer, 1985, Metallurgical Transactions A, Vol. 16, pp. 1005-1012.
26. *Evidence from x-ray and neutron powder diffraction patterns that the so-called icosahedral and decagonal quasicrystals of MnAl6 and other alloys are twinned cubic crystals*. **Pauling,**

- Linus**. s.l. : National Acad Sciences, 1987, Proceedings of the National Academy of Sciences, Vol. 84, pp. 3951-3953.
27. In Mysterious Pattern, Math and Nature Converge. *Wired*. [Online] <https://www.wired.com/2013/02/math-and-nature-universality/>.
28. **Baker, Alan**. *Transcendental number theory*. s.l. : Cambridge University Press, 1990. pp. 102-107.
29. *Tilings and patterns*. **Grunbaum, Branko and Shephard, G.-C.** s.l. : WH Freeman and Company, Co-published by Akademie-Verlag, 1987.
30. *Introduction to the theory of moduli*. **Mumford, David and Suominen, Kalevi**. 1970.
31. **Deutsch, David**. *The fabric of reality*. s.l. : Penguin UK, 1998.
32. **Fredkin, E**. Digital Mechanics, an Informational Process Based on Reversible Universal Cellular Automata. Cellular Automata, Theory and Experiment, H. Gutowitz (Eds) MIT. s.l. : North-Holland, p254-270, Amsterdam, 1990.
33. *Cellular automata as an alternative to (rather than an approximation of) differential equations in modeling physics*. **Toffoli, Tommaso**. s.l. : Elsevier, 1984, Physica D: Nonlinear Phenomena, Vol. 10, pp. 117-127.
34. **Wolfram, Stephen**. *A new kind of science*. s.l. : Wolfram media Champaign, 2002. Vol. 5.
35. *Information, physics, quantum: The search for links*. **Wheeler, John Archibald**. 1990.
36. **Rojas, Raúl**. *Neural networks: a systematic introduction*. s.l. : Springer Science & Business Media, 2013.
37. *The free will theorem*. **J Conway, S Kochen**. s.l. : Foundations of Physics, Springer, 2006.
38. **de Maupertuis, Pierre-Louis Moreau**. *Accord de différentes loix de la nature qui avaient jusqu'ici paru incompatibles*. 1744.
39. *Invariante variationsprobleme*. **Noether, Emmy**. 1918, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, mathematisch-physikalische Klasse, Vol. 1918, pp. 235-257.
40. *Heterotic Supergravity with Internal Almost-Kähler Configurations and Gauge $SO(32)$, or $E_8 \times E_8$, Instantons*. **Bubuianu, Laurențiu, Klee Irwin, and Sergiu I. Vacaru**. <https://arxiv.org/abs/1611.00223> : Submitted for publishing, 2016.
41. *Starobinsky Inflation and Dark Energy and Dark Matter Effects from Quasicrystal Like Spacetime Structures*. **Aschheim, R., Bubuianu, L., Fang, F., Irwin, K., Ruchin, V., & Vacaru, S**. <https://arxiv.org/pdf/1611.04858.pdf> : Manuscript submitted for publication, 2016.
42. *Anamorphic Quasiperiodic Universes in Modified and Einstein Gravity with Loop Quantum Gravity Corrections*. **Amaral, Marcelo M and Aschheim, Raymond and Bubuianu, Laurențiu and Irwin, Klee and Vacaru, Sergiu I and Woolridge, Daniel**. <https://arxiv.org/pdf/1611.05295.pdf> : Submitted for publication, 2016.
43. *A highly symmetric four-dimensional quasicrystal*. **Elser, Veit and Sloane, Neil J. A.** s.l. : IOP Publishing, 1987, Journal of Physics A: Mathematical and General, Vol. 20, p. 6161.
44. *Zur theorie der fastperiodischen funktionen*. **Bohr, Harald**. s.l. : Springer, 1925, Acta Mathematica, Vol. 46, pp. 101-214.
45. *Hydrodynamics of icosahedral quasicrystals*. **Lubensky, T. C., Ramaswamy, Sriram and Toner, John**. s.l. : APS, 1985, Physical Review B, Vol. 32, p. 7444.
46. *Reconstructing the universe*. **Ambjørn, Jan, Jurkiewicz, Jerzy and Loll, Renate**. s.l. : APS, 2005, Physical Review D, Vol. 72, p. 064014.
47. **Oriti, Daniele**. *Approaches to quantum gravity: Toward a new understanding of space, time and matter*. s.l. : Cambridge University Press, 2009.

48. *Quasicrystals. I. Definition and structure.* **Levine, Dov and Steinhardt, Paul J.** s.l. : APS, 1986, Physical Review B, Vol. 34, p. 596.
49. **Voss, Richard F.** Fractals in nature: from characterization to simulation. *The science of fractal images.* s.l. : Springer, 1988, pp. 21-70.
50. **Rosenblum, Bruce and Kuttner, Fred.** *Quantum enigma: Physics encounters consciousness.* s.l. : Oxford University Press, 2011.
51. *Robust universal complete codes for transmission and compression.* **Fraenkel, Aviezri S. and Kleinb, Shmuel T.** s.l. : Elsevier, 1996, Discrete Applied Mathematics, Vol. 64, pp. 31-55.
52. **Schrodinger, Erwin.** *What is Life?: The Physical Aspects of Living Cell with Mind and Matter and Autobiographical Sketches.* s.l. : Cambridge University Press, 1967.
53. *Codon populations in single-stranded whole human genome DNA are fractal and fine-tuned by the Golden Ratio 1.618.* **Perez, Jean-Claude.** s.l. : Springer, 2010, Interdisciplinary Sciences: Computational Life Sciences, Vol. 2, pp. 228-240.
54. *Topological quantum hashing with the icosahedral group.* **Burrello, Michele, et al., et al.** s.l. : APS, 2010, Physical review letters, Vol. 104, p. 160502.
55. **Coxeter, Harold Scott Macdonald.** *Regular polytopes.* s.l. : Courier Corporation, 1973.
56. *Double-slit quantum eraser.* **Walborn, S. P., et al., et al.** s.l. : APS, 2002, Physical Review A, Vol. 65, p. 033818.
57. *Feeling the future: A meta-analysis of 90 experiments on the anomalous anticipation of random future events.* **Bem, Daryl, et al., et al.** s.l. : Faculty of 1000 Ltd, 2015, F1000Research, Vol. 4.
58. *Relating doubly-even error-correcting codes, graphs, and irreducible representations of N-extended supersymmetry.* **Doran, C. F., et al., et al.** 2008, arXiv preprint arXiv:0806.0051.
59. *Adinkras: A graphical technology for supersymmetric representation theory.* **Faux, Michael and Gates, S. J.** s.l. : APS, 2005, Physical Review D, Vol. 71, p. 065002.
60. **Deutsch, David.** *The fabric of reality.* s.l. : Penguin UK, 1998.
61. **BEAUJARD, ROGER.** ERROR CORRECTION IS IN EVERYTHING. [Online] <http://rogerbeaujard.com/2012/09/error-correction-is-in-everything/#more-130>.
62. *The large-N limit of superconformal field theories and supergravity.* **Maldacena, Juan.** s.l. : Springer, 1999, International journal of theoretical physics, Vol. 38, pp. 1113-1133.
63. *Thermodynamic phase transitions of Kerr-Newman black holes in de Sitter space.* **Davies, Paul C. W.** s.l. : IOP Publishing, 1989, Classical and Quantum Gravity, Vol. 6, p. 1909.
64. *Entropic motion in loop quantum gravity.* **García-Islas, J. Manuel.** s.l. : NRC Research Press, 2016, Canadian Journal of Physics, Vol. 94, pp. 569-573.
65. *Nonlocality for two particles without inequalities for almost all entangled states.* **Hardy, Lucien.** s.l. : APS, 1993, Physical Review Letters, Vol. 71, p. 1665.
66. *Quantum criticality in an Ising chain: experimental evidence for emergent E8 symmetry.* **Coldea, R., et al., et al.** s.l. : American Association for the Advancement of Science, 2010, Science, Vol. 327, pp. 177-180.
67. *Golden ratio in quantum mechanics.* **Xu, Lan and Zhong, Ting.** s.l. : Citeseer, 2011, Nonlinear Sci. Lett. B, Vol. 1, pp. 10-11.
68. *Golden ratio discovered in a quantum world.* **Coldea, R., et al., et al.** 2010, Science, Vol. 8, pp. 177-180.
69. *Solid-state physics: Golden ratio seen in a magnet.* **Affleck, Ian.** s.l. : Nature Publishing Group, 2010, Nature, Vol. 464, pp. 362-363.

70. **Connes, Alain.** Noncommutative geometry year 2000. *Visions in Mathematics*. s.l. : Springer, 2000, pp. 481-559.
71. *Superstrings, Knots, and Noncommutative Geometry in E-Infinity Space.* **El Naschie, M. S.** s.l. : Springer, 1998, International journal of theoretical physics, Vol. 37, pp. 2935-2951.
72. *The theory of Cantorian spacetime and high energy particle physics (an informal review).* **El Naschie, M. S.** s.l. : Elsevier, 2009, Chaos, Solitons & Fractals, Vol. 41, pp. 2635-2646.
73. *Random matrices in physics.* **Wigner, Eugene P.** s.l. : SIAM, 1967, SIAM review, Vol. 9, pp. 1-23.
74. *The statistical properties of the city transport in Cuernavaca (Mexico) and random matrix ensembles.* **Krbálek, Milan and Seba, Petr.** s.l. : IOP Publishing, 2000, Journal of Physics A: Mathematical and General, Vol. 33, p. L229.
75. *Universal multifractal scaling of synthetic aperture radar images of sea-ice.* **Falco, Tony, et al., et al.** s.l. : IEEE, 1996, IEEE transactions on geoscience and remote sensing, Vol. 34, pp. 906-914.
76. *The unreasonable effectiveness of mathematics in the natural sciences.* **Wigner, Eugene.** s.l. : Thomson Higher Education, 1984, Mathematics: People, Problems, Results, Vol. 3, p. 116.
77. *A consistent approach toward atomic radii.* **Suresh, C. H. and Koga, Nobuaki.** s.l. : ACS Publications, 2001, The Journal of Physical Chemistry A, Vol. 105, pp. 5940-5944.
78. *Golden Sections of Interatomic Distances as Exact Ionic Radii and Additivity of Atomic and Ionic Radii in Chemical Bonds.* **Heyrovska, Raji.** 2009, arXiv preprint arXiv:0902.1184.
79. **Trebin, Hans-Rainer.** *Quasicrystals: structure and physical properties.* s.l. : John Wiley & Sons, 2006. pp. 212-221.
80. **Wolchover, Natalie.** In Mysterious Pattern, Math and Nature Converge. [ed.] N. E. W. S. SIMONS SCIENCE. June 2013.
81. *Periodic modification of the Boerdijk-Coxeter helix (tetrahelix).* **Sadler, Garrett, et al., et al.** 2013, arXiv preprint arXiv:1302.1174.
82. *The Sum of Squares Law.* **Kovacs, Julio, et al., et al.** 2012, arXiv preprint arXiv:1210.1446.
83. *Law of Sums of the Squares of Areas, Volumes and Hyper-Volumes of Regular Polytopes from Clifford Algebras.* **Perelman, Carlos Castro, Fang, Fang and Irwin, Klee.** s.l. : Springer, 2013, Advances in Applied Clifford Algebras, Vol. 23, pp. 815-824.
84. **Baruk, Ilija.** *Causality II: A theory of energy, time and space.* 2008.
85. *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?* **Einstein, Albert.** s.l. : Wiley Online Library, 1905, Annalen der Physik, Vol. 323, pp. 639-641.
86. **Griffiths, David Jeffery.** *Introduction to quantum mechanics.* s.l. : Pearson Education India, 2005.
87. *Is “the theory of everything” merely the ultimate ensemble theory?* **Tegmark, Max.** s.l. : Elsevier, 1998, Annals of Physics, Vol. 270, pp. 1-51.
88. **Born, Max, Einstein, Albert and Born, Irene.** *The Born Einstein Letters: correspondence between Albert Einstein and Max and Hedwig Born from 1916 to 1955 with commentaries by Max Born. Translated by Irene Born.* s.l. : Basingstoke, Macmillan Press, 1971.
89. *Opticks: or a treatise of the reflexions, refractions, inflexions and colours of light.* **Newton, Isaac.** 1704.
90. **De Broglie, Louis and de Broglie (Paris), Fondation Louis.** *Recherches sur la théorie des quanta.* s.l. : Masson Paris, 1963.
91. *Electron time, mass and zitter.* **Hestenes, David.** s.l. : Citeseer, 2008, The Nature of Time Essay Contest. Foundational Questions Institute.

92. *Question of parity conservation in weak interactions.* **Lee, Tsung-Dao and Yang, Chen-Ning.** s.l. : APS, 1956, Physical Review, Vol. 104, p. 254.
93. **Pruett, Charles David.** *Reason and wonder: A Copernican revolution in science and spirit.* s.l. : ABC-CLIO, 2012.
94. **Sullivan, J. W. N.** Interview with Erwin Schrödinger. 1 1931.
95. **Linde, Andrei.** *Chaos, Consciousness, and the Cosmos.* FQXI. 2011. Tech. rep. URL: <http://fqxi.org/community/articles/display/145>.
96. **Skrbina, David.** *The Metaphysics of Technology.* s.l. : Routledge, 2014. Vol. 94.
97. **Wilber, Ken.** *Quantum questions.* s.l. : Shambhala Publications, 2001.
98. *A new approach to the hard problem of consciousness: a quasicrystalline language of "primitive units of consciousness" in quantized spacetime (part I).* **Irwin, Klee.** 2014, Journal of Consciousness Exploration & Research, Vol. 5.
99. *Populating the landscape: A top-down approach.* **Hawking, Stephen W. and Hertog, Thomas.** s.l. : APS, 2006, Physical Review D, Vol. 73, p. 123527.
100. *Nonclassicality of Temporal Correlations.* **Brierley, Stephen, et al., et al.** s.l. : APS, 2015, Physical review letters, Vol. 115, p. 120404.