THE FIBONACCI ICOSAGRID AND THE ELSER SLOANE QC

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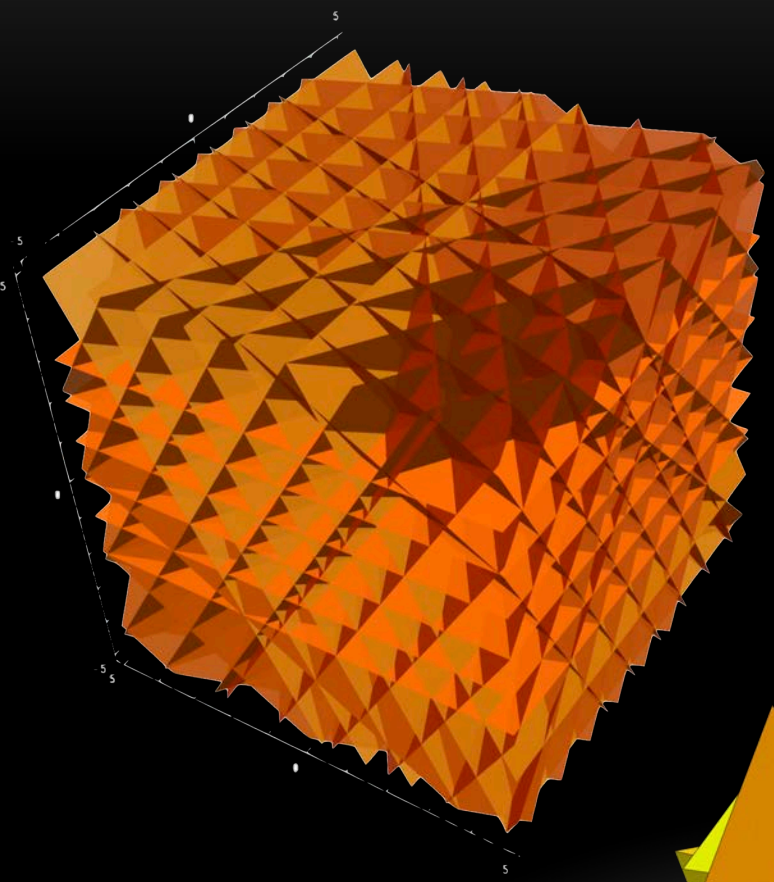
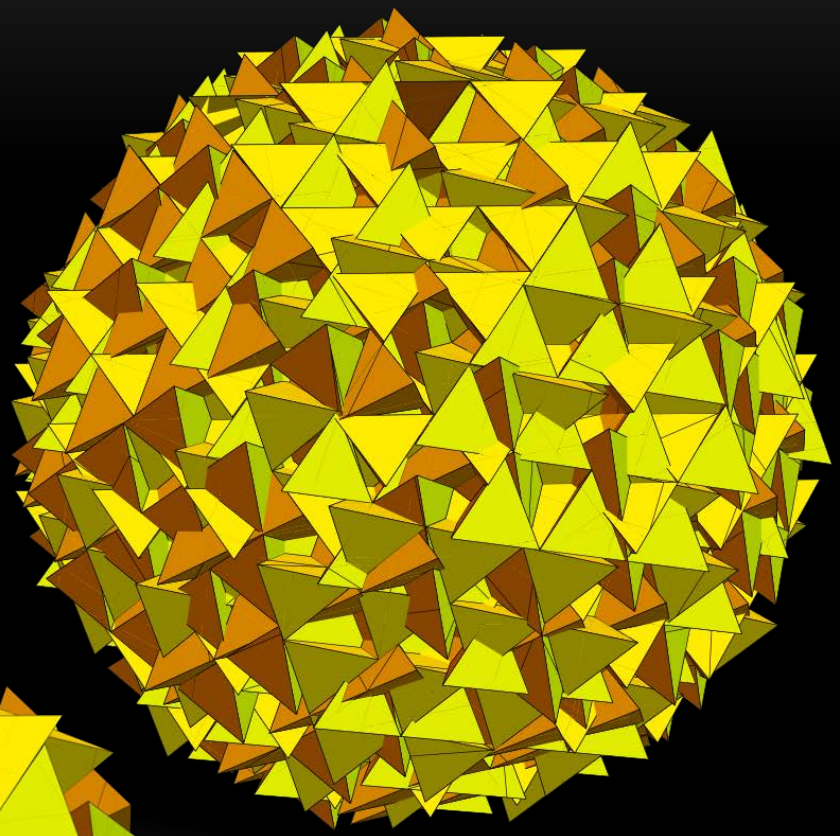
Quantum Gravity Research

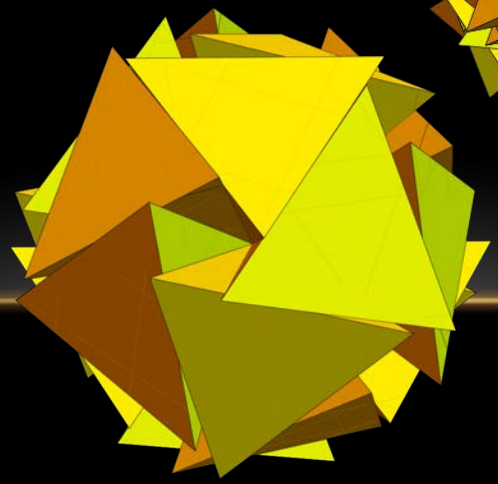
OUTLINE

- Icosagrid and Fibonacci Icosagrid
- The Esler Sloane QC, 600-cell and the 600-cell compound
- The 3D compound QC
- The matching between the Fibonacci IcosaGrid and the 3D compound QC
- Connection between the 20G twist and 600-cell compound

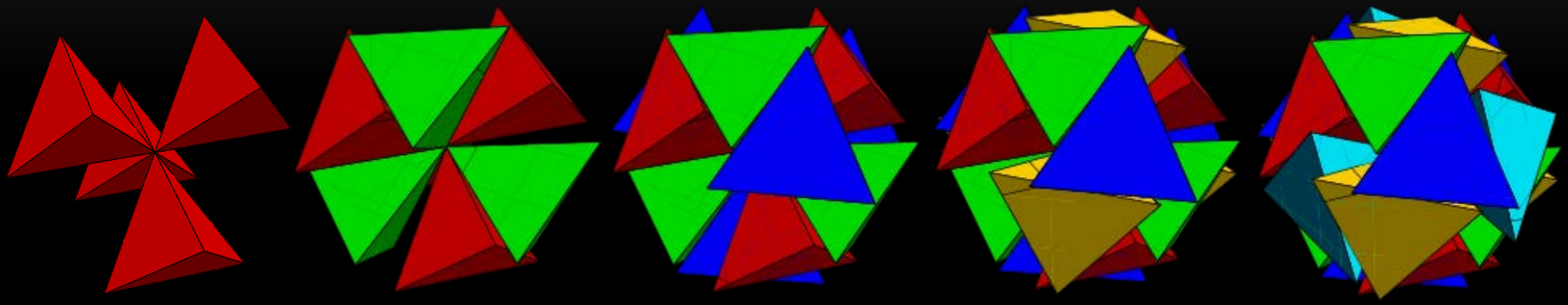
ICOSAGRID

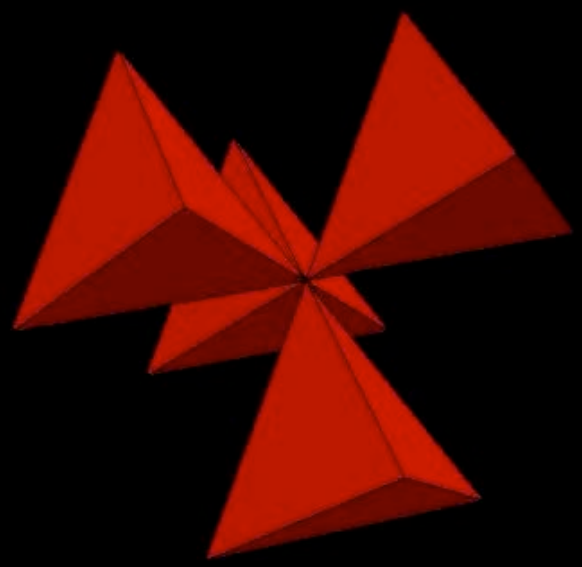




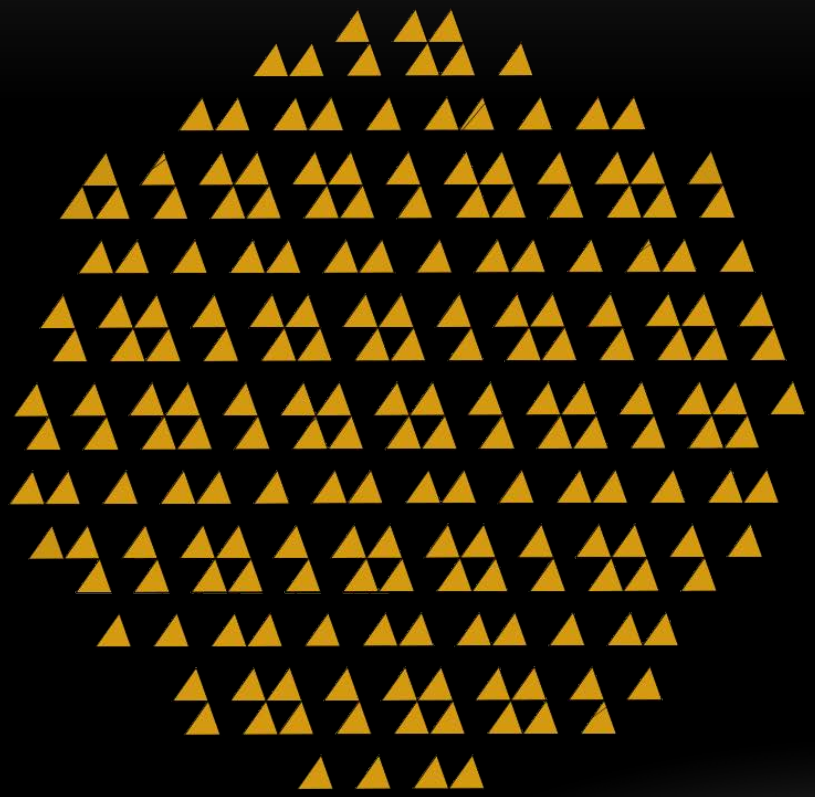


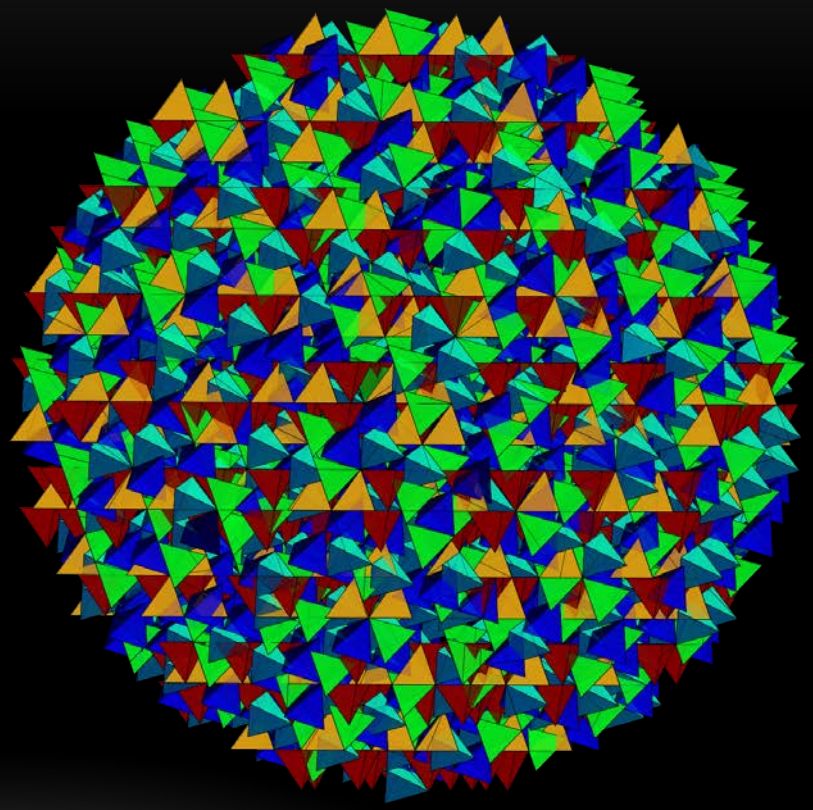
ANOTHER WAY OF BUILDING THE ICOSAGRID



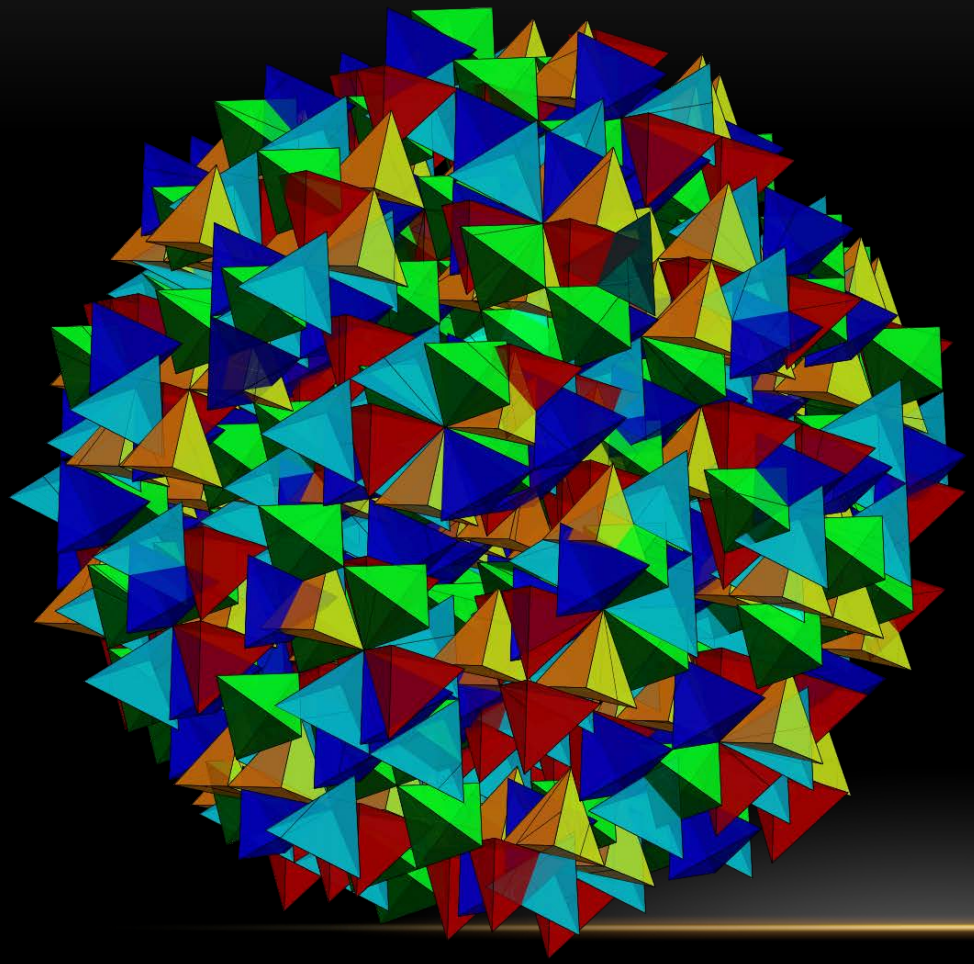


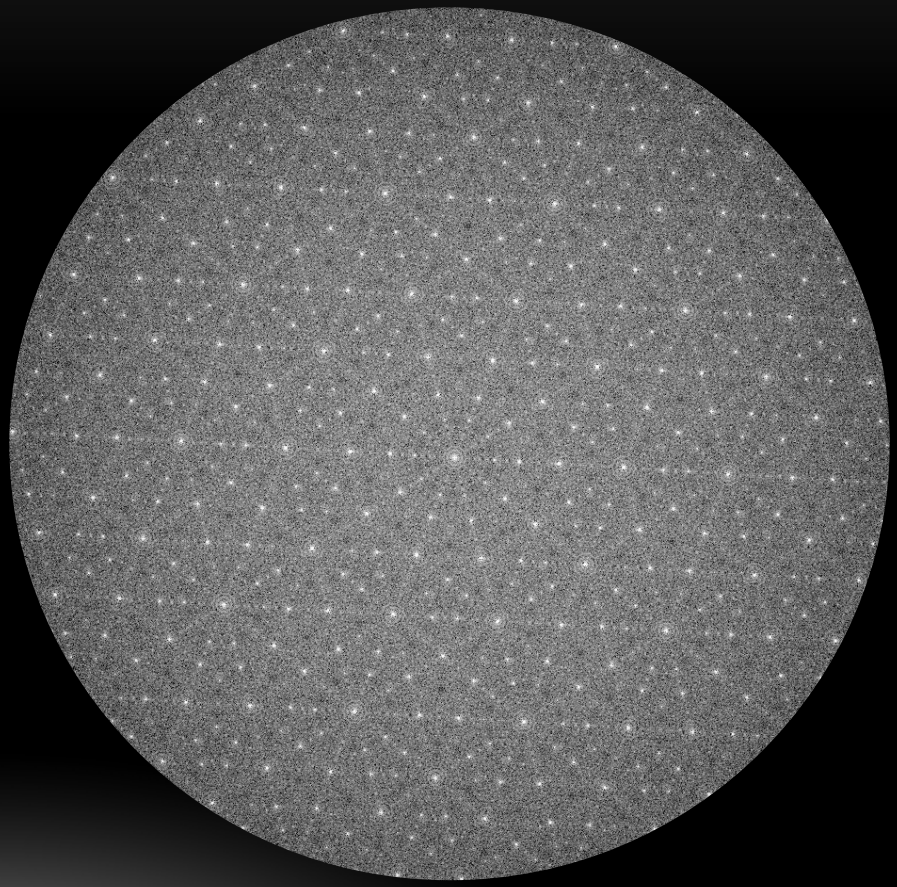
FIBONACCI ICOSAGRID





FIBONACCI ICOSAGRID





THE ELSER-SLOANE QC

The Elser-Sloane QC is constructed in real four-dimensional Euclidean space, having the noncrystallographic reflection group [3,3,5] of order 14400 as its point group. It is obtained as a projection of the eight-dimensional lattice E8, and has as a cross-section a three-dimensional quasicrystal with icosahedral symmetry.

Projection mapping matrix: $\Pi = -\frac{1}{\sqrt{5}}\begin{bmatrix} \sigma^{-1} I & H \\ H & \sigma I \end{bmatrix}$,

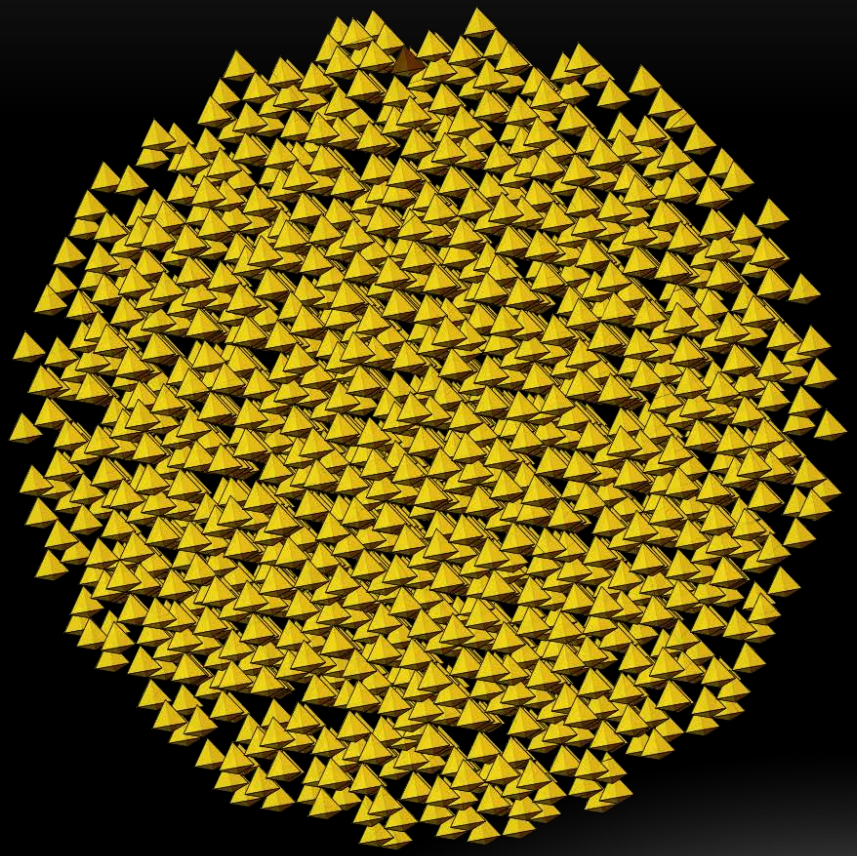
where $I = I_4 = \mathrm{diag}\{1,1,1,1\}$, $\sigma = \frac{\sqrt{5}-1}{2}$, $\mathrm{H} = \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$.

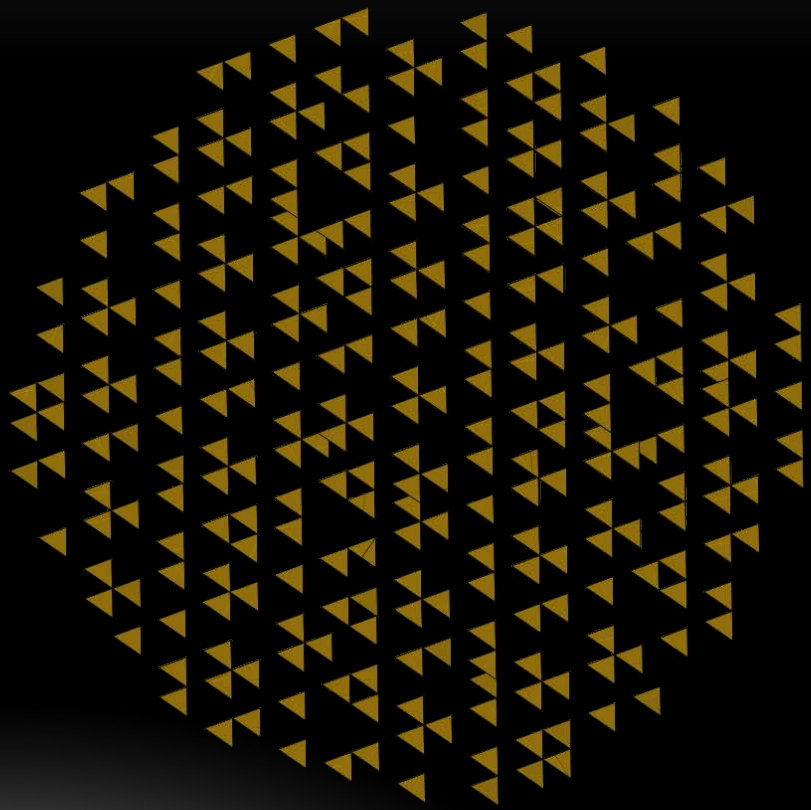
THE ELSER-SLOANE QC

Properties of this QC (Ref):

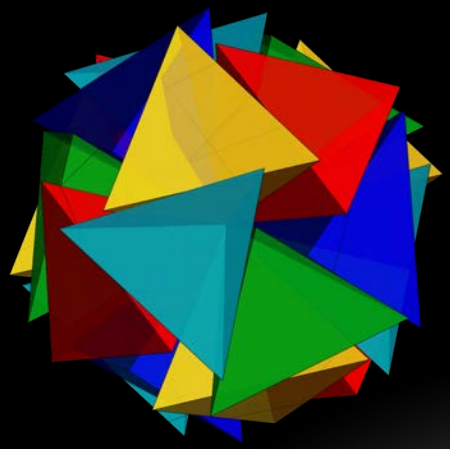
- It is invariant under a point group (fixing the origin) isomorphic to $G\,1 = [\,3\,,\,3\,,\,5\,]$.
- It is closed under multiplication by τ.
- It is a discrete set of points.
- It has a 600-cell {3,3,5} at its center, whose 120 vertices are mapped from 120 of the 240 minimal vectors of $E\,8$. Similarly exactly 120 of the 2160 vectors in $E\,8$ of length 2 mapped to a slightly larger (τ times) $\{\,3\,,\,3\,,\,5\,\}$ concentric with the first.
- It has a cross-section which is a 3-D quasicrystal with icosahedral symmetry.

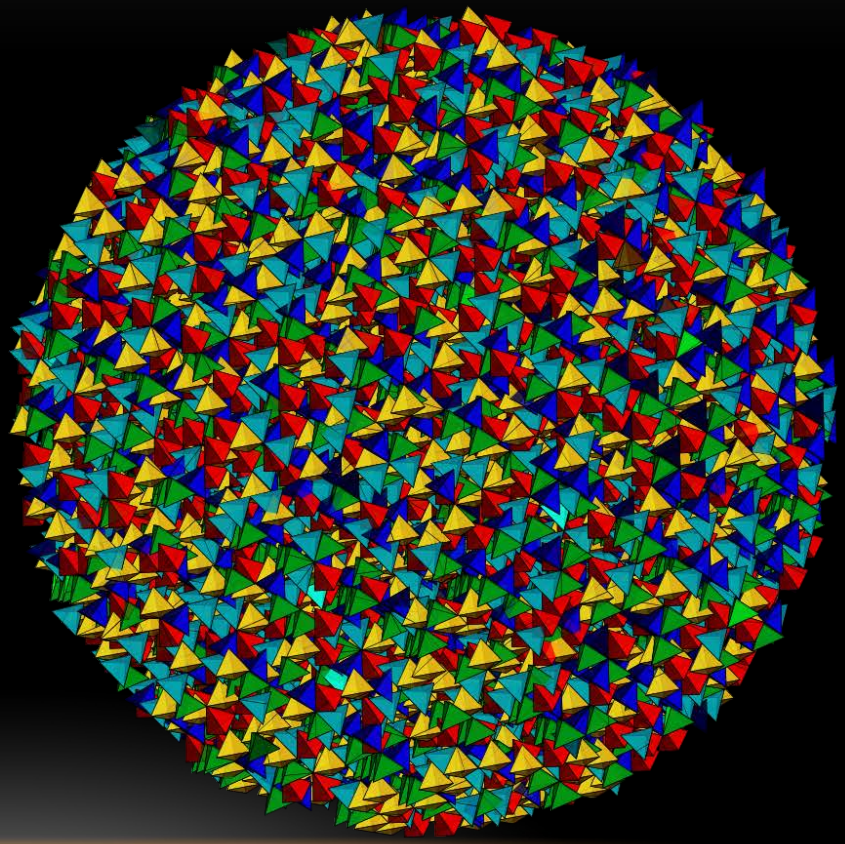
THE 3D COMPOUND QC – TYPE I



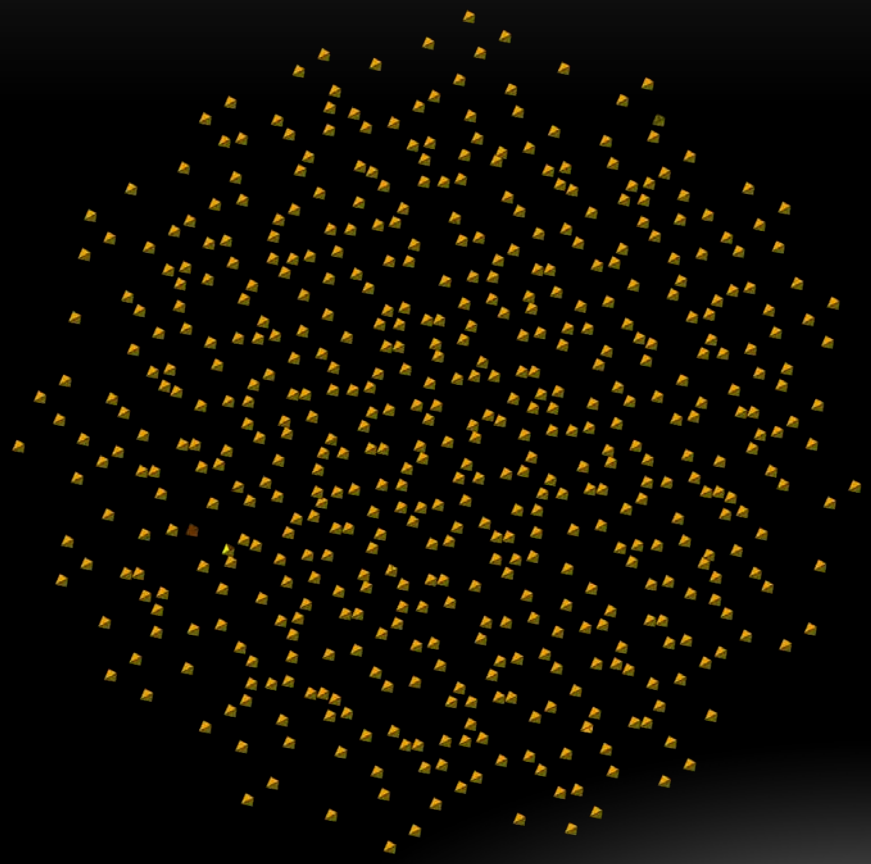


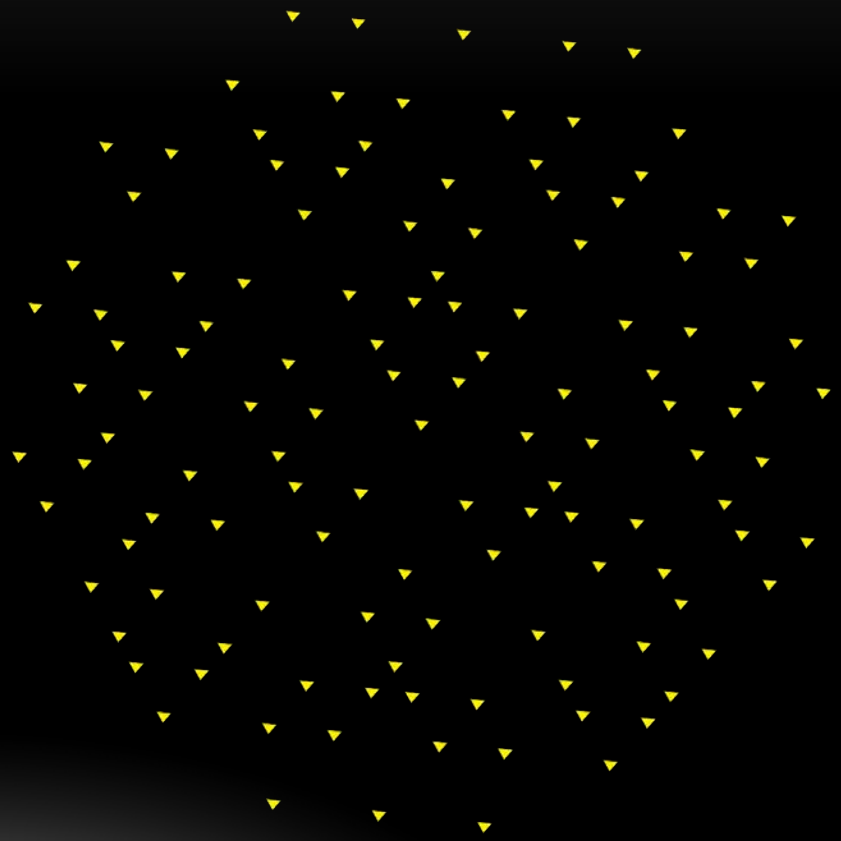
THE 3D COMPOUND QC – TYPE I



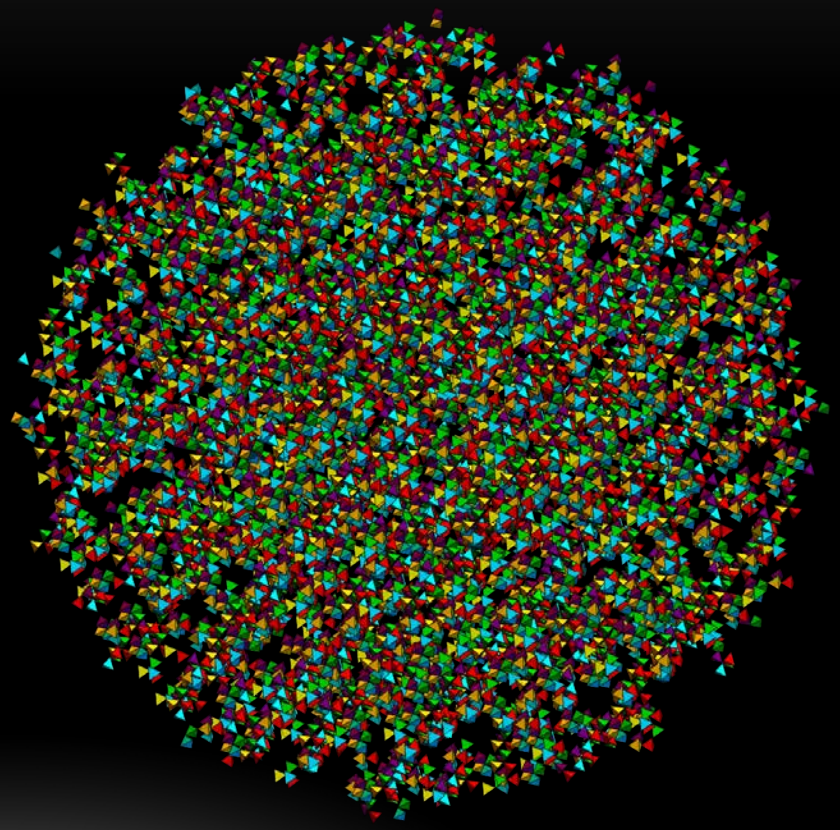


THE 3D COMPOUND QC – TYPE II





THE 3D COMPOUND QC – TYPE II



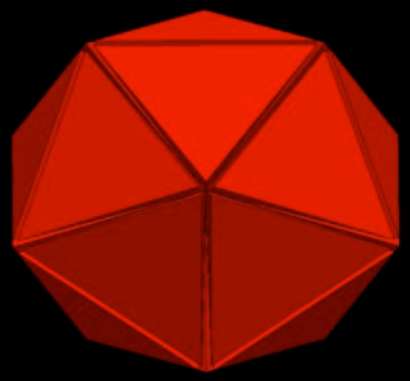
WHY COMPOSE THEM TOGETHER THIS WAY?

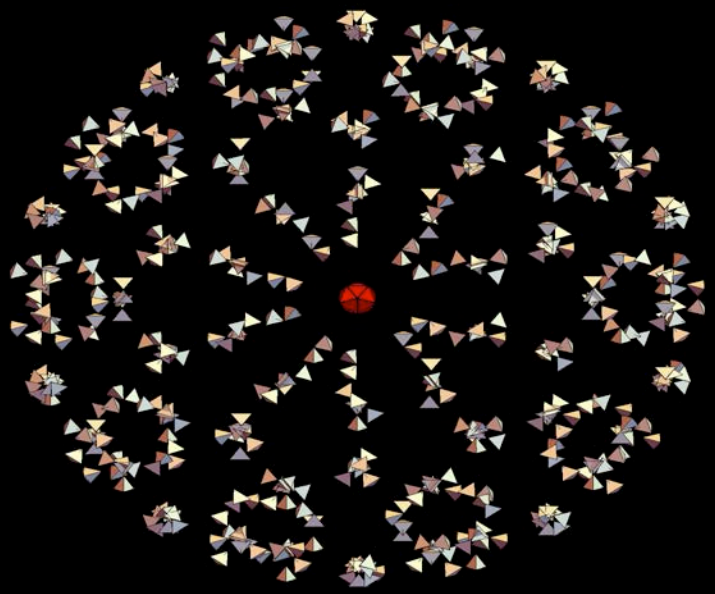
The open and closing animation of the 20Gs

Match to the Fibonacci IcosaGrid

Match to the 600-cell compound

THE GOLDEN TWIST AND THE CLOSING OF THE 20G TWISTS





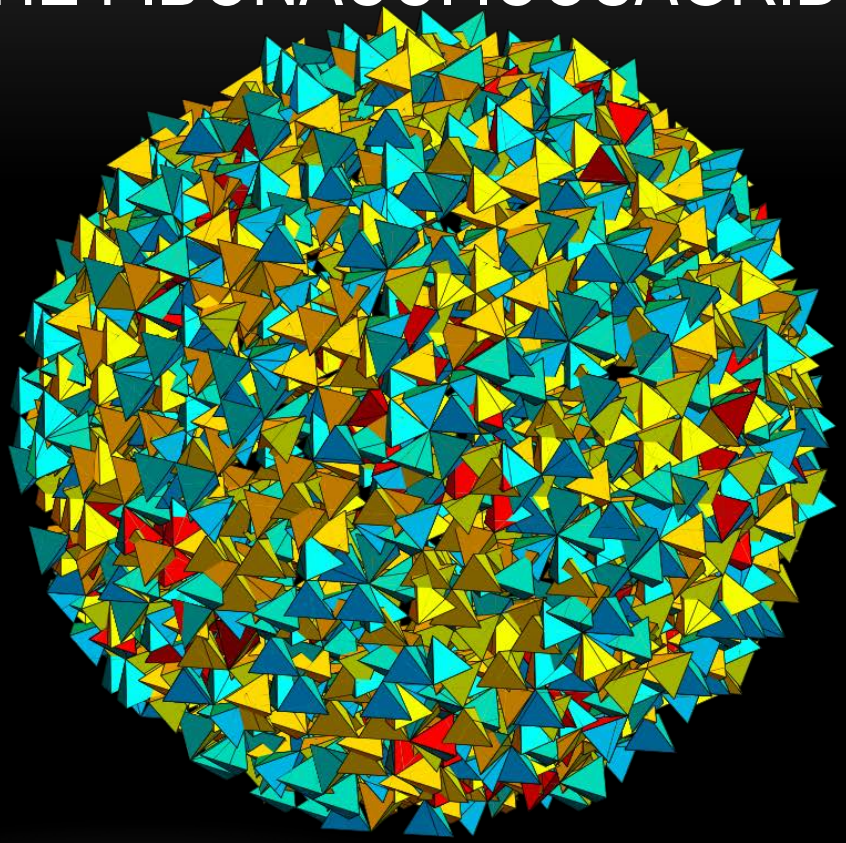
MAPPING BETWEEN THE 3D COMPOUND QC (TYPE I AND TYPE II) AND THE FIBONACCI ICOSAGRID

Fibonacci Icosagrid includes CMPD-QC type II

Fibonacci Icosagrid does not include CMPD-QC type I

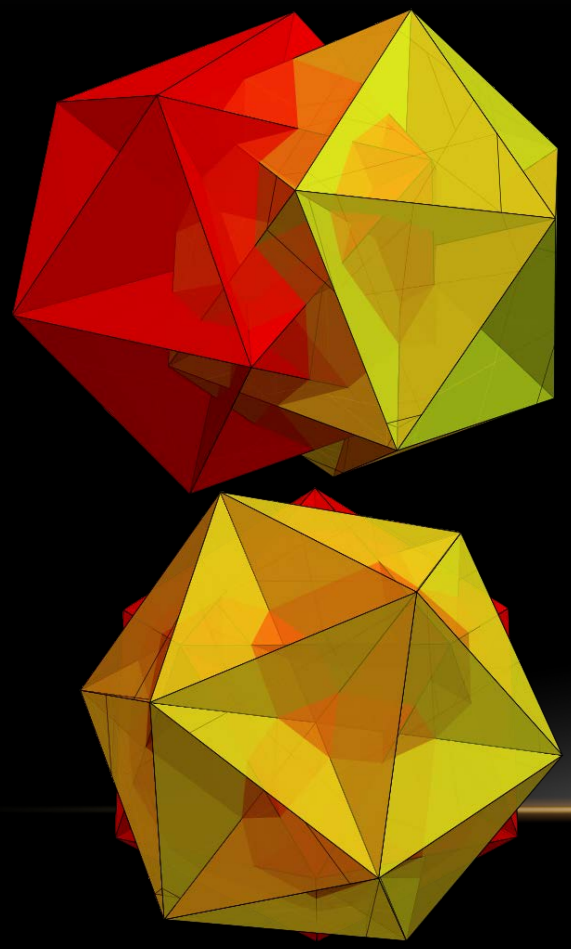
CMPD-QC type I includes CMPD-QC Type II

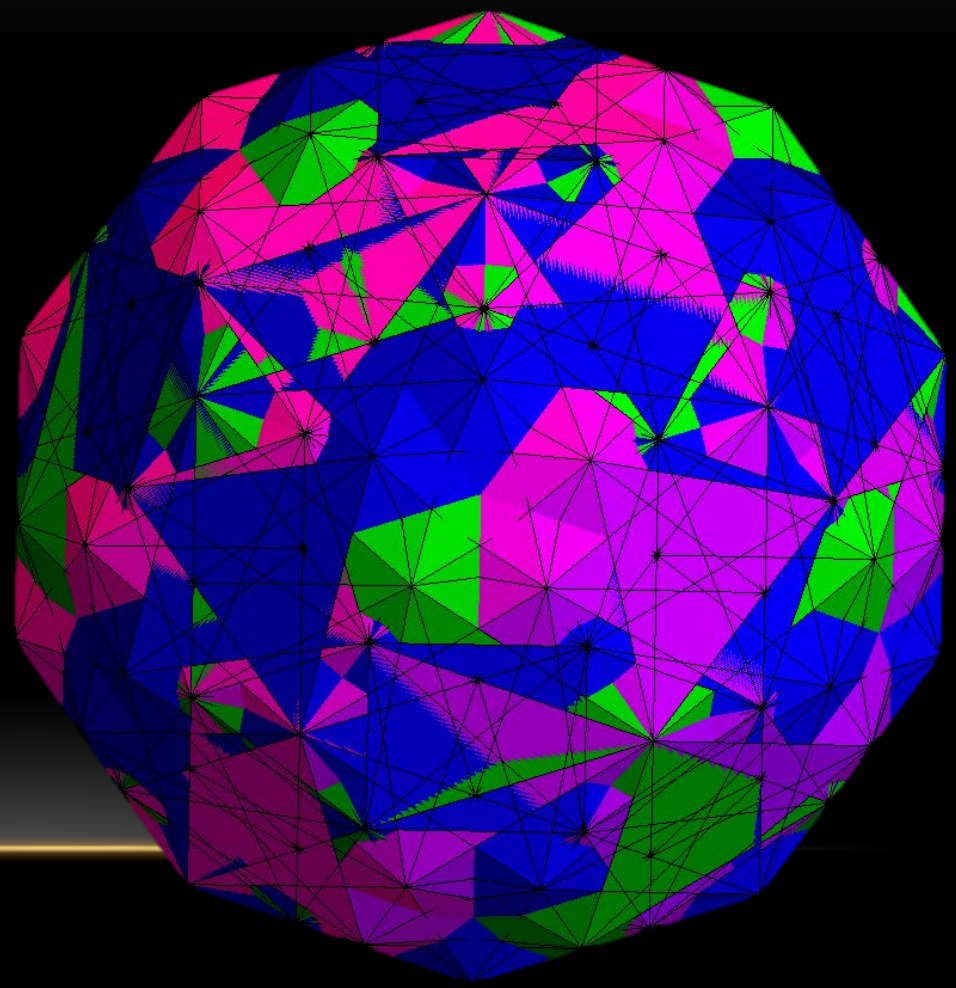
New Modified Fibonacci Icosagrid with Icosahedral symmetry should be able to include both CMPD-QC.



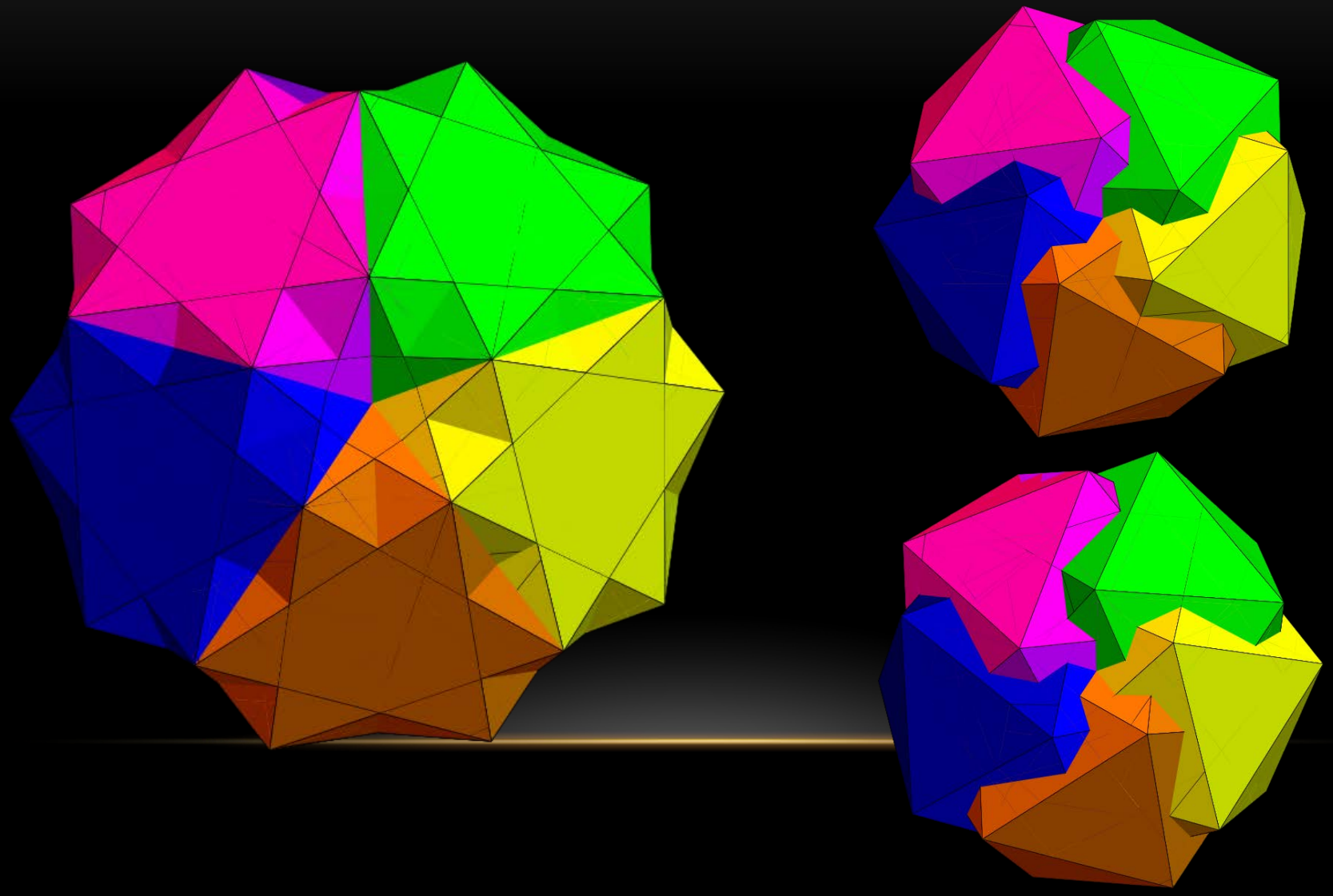
THE 600-CELL AND ITS COMPOUND (PROJECTION OF THE E8 VORONOI CELL)

Each vertex of the E8 Voronoi cell corresponds to one 7-facets in the Gosset 4_{21} polytope.

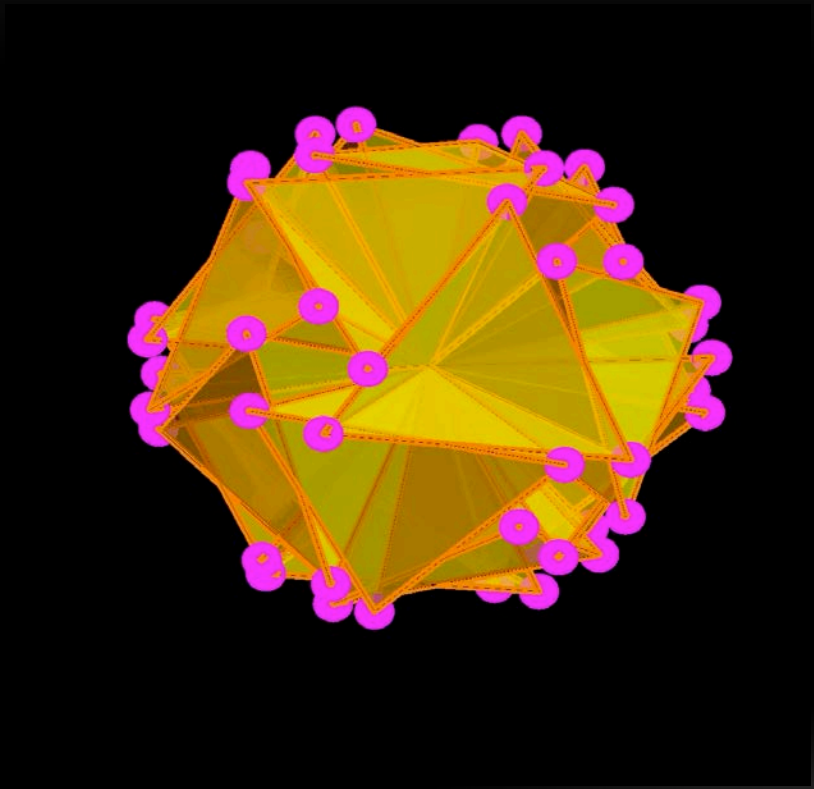


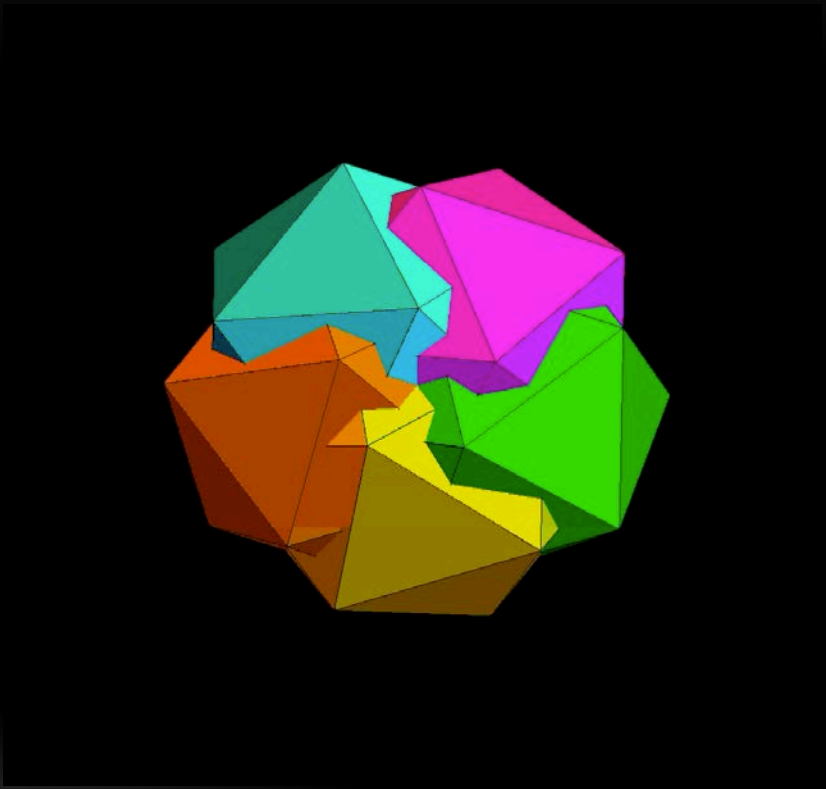


THE 600-CELL AND ITS COMPOUND (PROJECTION OF THE E8 VORONOI CELL)

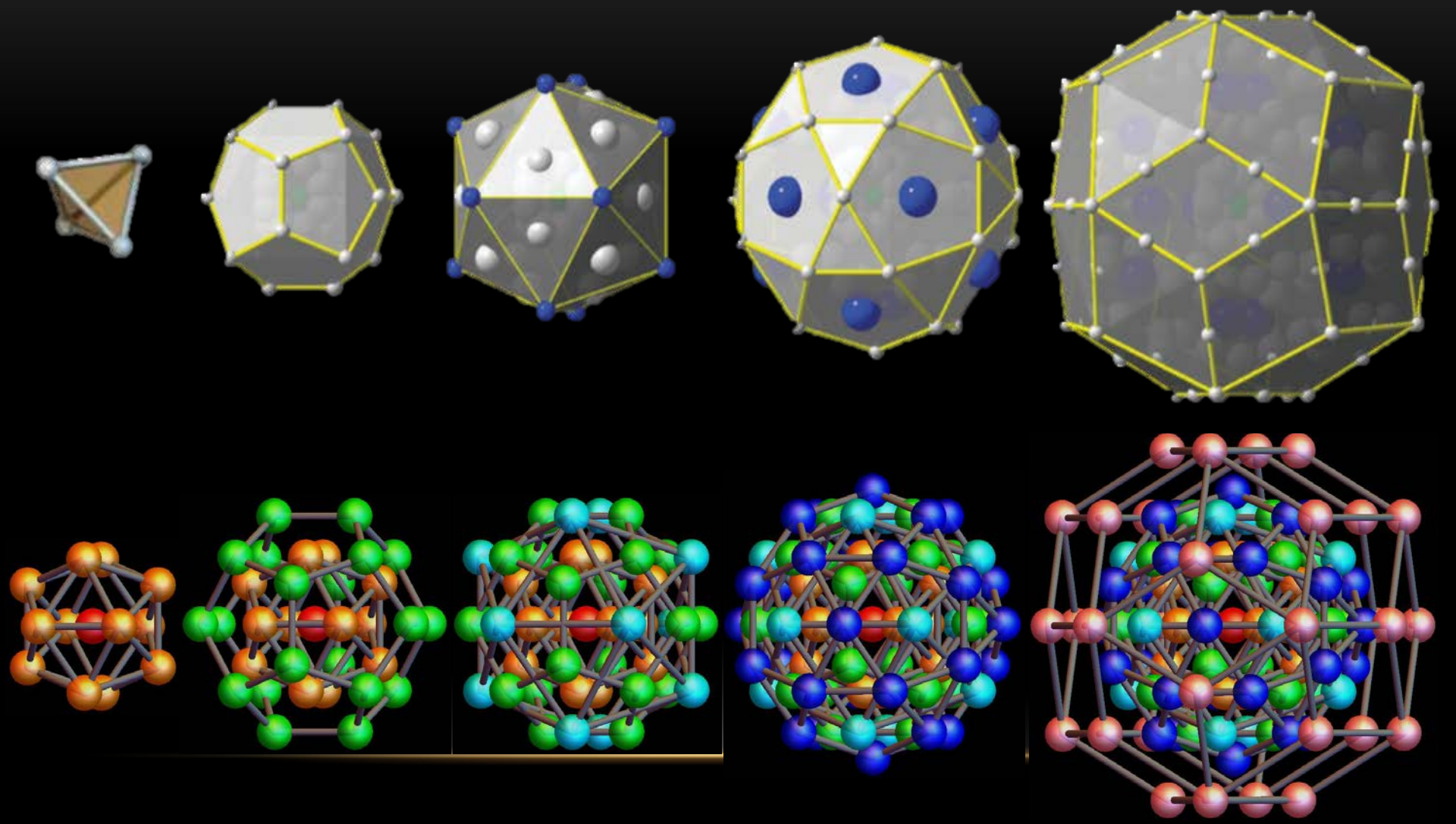


CONNECTION BETWEEN THE 20G TWIST （LEFT AND RIGHT SUPERPOSITION） AND 600-CELL COMPOUND

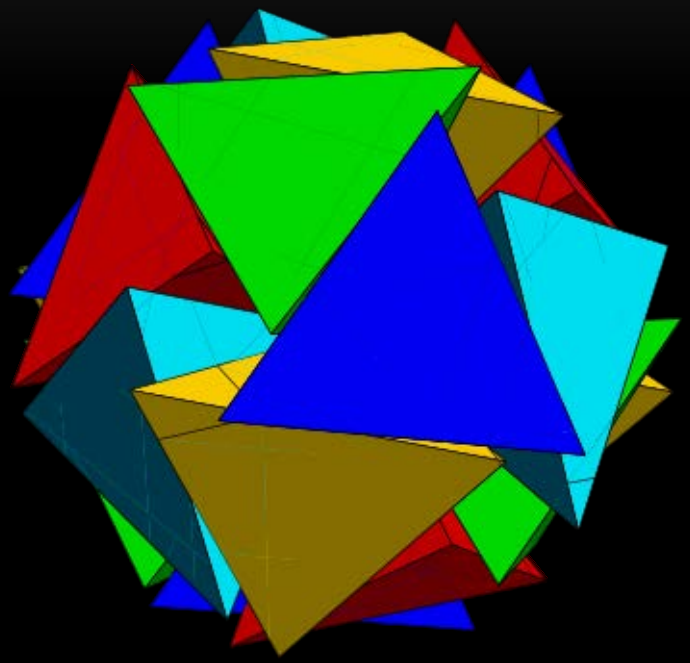




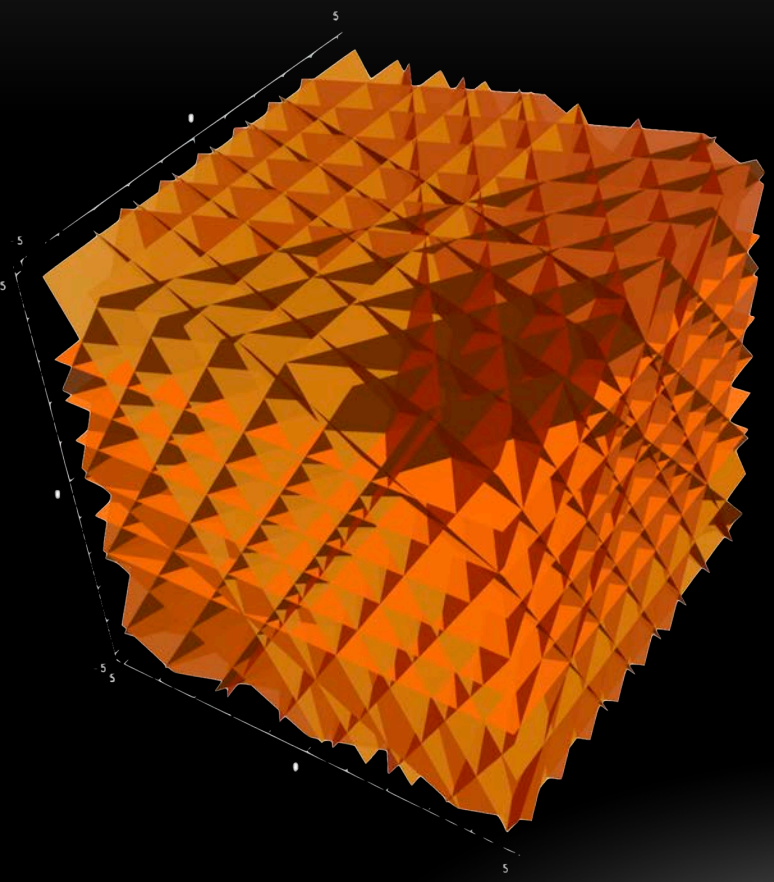
CONNECTION BETWEEN THE 3D PROJECTION OF THE ELSER-SLOANE QC AND THE TSAI-TYPE QC

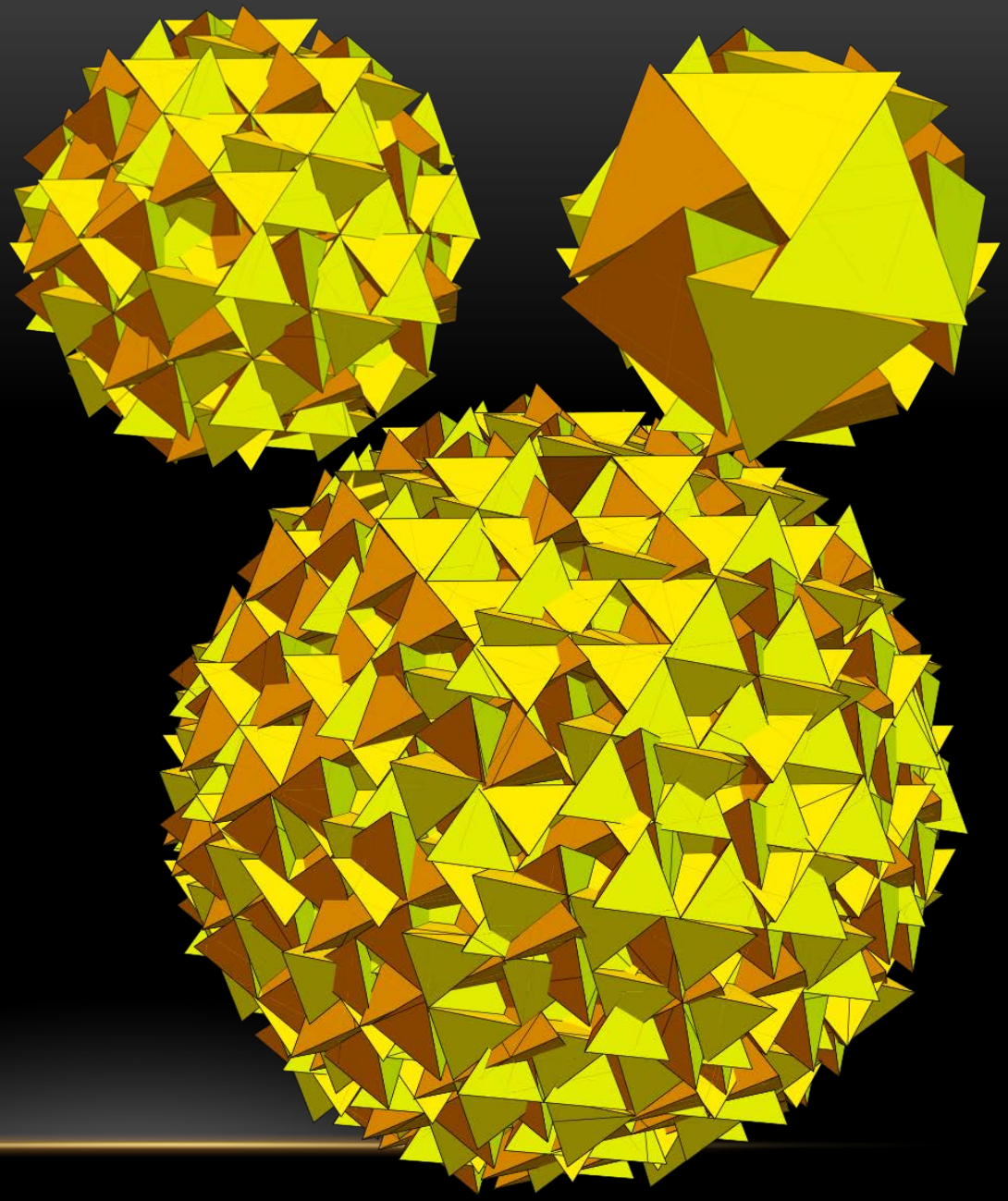


SUMMARY



ICOSAGRID





THE ELSER-SLOANE QC

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720-cell = 120 + 600: This is the orbit represented by $W(H_4)(0100)$ consisting of 1200 vertices. Its cells are made of 120 icosidodecahedra and 600 tetrahedra. At any vertex there join 3 icosidodecahedra and 2 tetrahedra whose centers are represented respectively by the vectors $\omega_4, r_3 r_4 \omega_4, (r_3 r_4)^2 \omega_4$ up to a scale factor and by the vectors $\omega_1, r_1 \omega_1$.

The dual polytope of the 720-cell consists of 720 vertices which is the union of the orbits $W(H_4)(0001) \oplus \frac{2}{3\tau} W(H_4)(1000)$.[26] Each cell of the dual polytope is a dipyramid with an equilateral triangular base. The dual polytope is the projection of the Voronoi cell of E_8 (see Ref. 27) with 19,440 vertices into 4D space. Another semi regular polytope of the Coxeter group $W(H_4)$ has 720 vertices and 720 cells which we denote by $720' - cell = 120 + 600$. The $720' - cell$ is represented by the orbit $W(H_4)(0010)$. It consists of the cells of 120 icosahedra and 600 octahedra and with this property it is the only Archimedean solid with the $W(H_4)$

$$-\frac{1}{\sqrt{5}}\begin{bmatrix} \sigma^{-1} I & H \\ H & \sigma I \end{bmatrix}$$

$$c = \frac{1}{\sqrt{4 + 2\sigma}} = 0.602\ldots, \quad H = \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix},$$

CONNECTION BETWEEN THE 3D PROJECTION OF THE ELSER-SLOANE QC AND THE TSAI-TYPE QC