Nonification

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Raymond Aschheim | Nonification
This talk is dedicated to the career of Piero Truini which opened the path to an exceptional an magic unification.

We shall see how the partition of the E8 lattice into three A8 lattices, inviting us to use 9 dimensions coordinates, guide also us to an exceptionally symmetric unification... without supersymmetry.

"It is amusing to speculate on the possibility of a theory based on E9." is the conclusion of the chapter V, Exceptional Unification, of Pr. Anthony Zee’s book.

REF:

Zee, A. Unity of forces in the universe World Scientific. 1982
An Simplex lattices are naturally expressed in $n + 1$ dimension coordinates satisfying $\sum_{k=1}^{n+1} x_k = 0$.
Introduction
9D coordinates

- $A_n$ Simplex lattices are naturally expressed in $n + 1$ dimension coordinates satisfying $\sum_{k=1}^{n+1} x_k = 0$
- $E_8$ lattice is the superposition of three $A_8$ lattices: $E_8 = \bigcup_{i=0}^{2} A_8^i$
- 72 of its roots are permutations of $\{3^1, -3^1, 0^7\}, \cong 0[3], \in 3A_8^0$
- 84 of its roots are $\mathcal{P}(-2^3, 1^6), \cong 1[3], \in 3A_8^1$
- 84 of its roots are $\mathcal{P}(2^3, -1^6), \cong 2[3], \in 3A_8^2$

$E_8 = \text{SU}(3) F + E_6 = A_2 F + E_6$
An Simplex lattices are naturally expressed in $n + 1$ dimension coordinates satisfying $\sum_{k=1}^{n+1} x_k = 0$

$E_8$ lattice is the superposition of three $A_8$ lattices: $E_8 = \bigcup_{i=0}^{2} A_8^i$

72 of its roots are permutations of $\{3^1, -3^1, 0^7\}$, $\cong \mathbf{O}[3]$, $\in 3A_8^0$

84 of its roots are $\mathbb{P}(-2^3, 1^6)$, $\cong 1[3]$, $\in 3A_8^1$

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\[
E_8 = SU(3)_F + E_6 = A_{2F} + E_6
\]

$248 \setminus A_{2F} + E_6 = (1, 78) + (8, 1) + (3, 27) + (\bar{3}, 2\bar{7})(1)$
Introduction
E6 subgroup in 9D

- $E_6$ lattice is the superposition of three $A_8$ lattices satisfying
  \[ \sum_{k=1}^{3} x_k = \sum_{k=4}^{6} x_k = \sum_{k=7}^{9} x_k = 0. \]
- 18 of its roots are $\mathcal{P}(3^1, -3^1, 0^7), \cong O[3], \in 3A_8^0$

Figure: 3 orthogonal $A_2$ in $E_6$; from left to right: (a) $A_2L$ (b) $A_2C$ (c) $A_2R$. 
Introduction
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  \[ \sum_{k=1}^{3} x_k = \sum_{k=4}^{6} x_k = \sum_{k=7}^{9} x_k = 0. \]
- 18 of its roots are $\mathbf{3}(3^1, -3^1, 0^7), \cong o[3], \in 3A_8^0$
- 27 of its roots are $\mathbf{3}(−2^3, 1^6), \cong 1[3], \in 3A_8^1$

Figure: 27 lepto-quarks bosons B from $E6 \cap A_8^1$;
**Introduction**

E6 subgroup in 9D

- **E6 lattice** is the superposition of three $A_8$ lattices satisfying
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![Diagram of E6 lattice](image)

**Figure:** 27 anti-lepto-quarks bosons $\hat{B}$ from $E6 \cap A_8^2$;
Introduction

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\[ E_6 = SU(3)_L + SU(3)_C + SU(3)_R + B + \overline{B} \quad (2) \]

\[ 78 = (8, 1, 1) + (1, 8, 1) + (1, 1, 8) + (3, 3, 3) + (\bar{3}, \bar{3}, \bar{3}) = B_L + B_C + B_R + B + \overline{B} \quad (3) \]
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\[ 27 = (3, \overline{3}, 1) + (1, 3, \overline{3}) + (\overline{3}, 1, 3) = q^\gamma + \hat{q}^\beta + l^\alpha \quad (4) \]
Introduction

E6 subgroup in 9D

- $E_6$ lattice is the superposition of three $A_8$ lattices satisfying
  \[ \sum_{k=1}^{3} x_k = \sum_{k=4}^{6} x_k = \sum_{k=7}^{9} x_k = 0. \]
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\[ E_6 = SU(3)_L + SU(3)_C + SU(3)_R + B + \bar{B} \]  \hspace{1cm} (2)

\[ 78 = (8, 1, 1) + (1, 8, 1) + (1, 1, 8) + (3, 3, 3) + (\bar{3}, \bar{3}, \bar{3}) = B_L + B_C + B_R + B + \bar{B} \]  \hspace{1cm} (3)

\[ 27 = (3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3) = q^\gamma_\alpha + \hat{q}^\beta_\gamma + I^\alpha_\beta \]  \hspace{1cm} (4)

- $q^\gamma_\alpha = \begin{bmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ h_r & h_g & h_b \end{bmatrix}$
- $\hat{q}^\beta_\gamma = \begin{bmatrix} \hat{u}_c & \hat{d}_c & \hat{h}_c \\ \hat{u}_m & \hat{d}_m & \hat{h}_m \\ \hat{u}_y & \hat{d}_y & \hat{h}_y \end{bmatrix}$
- $I^\alpha_\beta = \begin{bmatrix} N_1 & E^- & e^- \\ E^+ & N_2 & \nu_e \\ e^+ & \hat{\nu}_e & N_3 \end{bmatrix}$  \hspace{1cm} (5)
The relationship between the $E_8$ lattice and the Simplex lattice, $E_8 = 3A_8$, is illustrated and has been extended to exceptional periodicity algebras $^{[oh,oi]}$.

- exceptionally $84 = \Lambda^3 \mathbb{C}^9$ 3-form and $\overline{84} = \Lambda^6 \mathbb{C}^9$ 6-form in $SU(9)$ $^{[oi]}$.
- or generally $84 = 28 + 56 = \Lambda^2 \mathbb{C}^8 \bigoplus \Lambda^3 \mathbb{C}^8$ 2-form and 3-form, and $\overline{84} = 56 + 28 = \Lambda^6 \mathbb{C}^8 \bigoplus \Lambda^5 \mathbb{C}^8$ 6-form and 5-form in $Cl(8)$.

**REF:**

A8 extension of the standard model
A8 includes A2L, A2C and A2R

\[ 27 = (3, \bar{3}, 1) + (1, 3, 3) + (\bar{3}, 1, 3) \]  \hspace{1em} (7)

27 breaks under SU(3)_C x SU(2)_L x U(1)_Y as

\[ 27 = 2(1, 1, 0) + (1, 2, \frac{1}{2}) + (3, 2, -\frac{1}{3}) + 2(1, 2, -\frac{1}{2}) \]
\[ + 2(3, 1, -\frac{1}{3}) + (1, 1, 1) + (3, 1, -\frac{2}{3}) + (3, 2, \frac{1}{6}) \]  \hspace{1em} (8)
Tensor Network

- 248D algebra $E_8$ is coded by $G_{\pm} \in \mathcal{S}(\mathbb{O})$, $H_1, \ldots, H_7, H_+, H_- \in \text{Tr}_o(\mathcal{M}_8^3)$
- Its action on $J \in \mathcal{M}_8^3 \otimes \mathcal{M}_8^3$ is

$$E_8(H_1, \ldots, H_-)(J) = \delta J = [H_+, \mathcal{R}(J), H_-] - \sum_{k=1}^{7} e^{G_+} e_k e^{G_-} H_k \cdot \mathcal{R}(e_k J)$$

$$T = \sum_{j_i, j_j, o_i, o_j, z_i, z_j=1}^3 T^{j_i}_{j_j} o_i^{z_i} j_i^{o_j} o_j z_i j_j o_j z_j$$

$$T = J_1^1 \ J_2^1 \ J_3^1, \quad J_1^o \ J_2^o \ J_3^o, \quad J_{m_j}^{10} = O_1^{o_1} O_2^{o_2} O_3^{o_3}, \quad O_j^{10} = z_1^{10} z_2^{10} z_3^{10}$$

Figure: Fibonacci spaced tensor network projection
We insert the standard model in a spinfoam by a fermionic quantum tetrahedron whose 4 vertices have SU(3) values coming from the 4 A2 of E8.

\[
Z = \sum_{\{c\}} \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_{ve} \int_{SU(2)} dh_{ef} \int_G dU_{ve} \prod_f d_{j_f} \chi^{\gamma_j_{j_f}j_f} \left( \prod_{e \in \partial_f} \left( g_{se, h_{ef} g_{et_e}^{-1}} \right)^{\epsilon_{ef}} \right) \prod_{e \in \partial_f} \chi^{j_f} (h_{ef}) \prod_c (-1)^{|e|} \chi^{\frac{1}{2}} \left( \prod_{e \in c} \left( g_{se, U_{se} U_{et_e}^\dagger g_{et_e}} \right)^{\epsilon_{ee}} \right).
\]

Figure: Partition function with fermion

Integral on cycles will reduce to SU(3) six-j symbols, when edges SU(2) are embedded in SU(3).
Magic star\(^{[ok]}\) projected\(^{[ol]}\) from Gosset polytope

**REFERENCES:**

Jordan Matrix:

- Each $E_8$ vertex holds an exceptional Jordan matrix $J \in M_8^3$
- 10D Minkowski Spacetime with a transversal octonion $o$ as $J_2 = \left( \begin{array}{cc} t-x_8 & \bar{o} = x^0 e_0 - \sum_{k=1}^7 x^k e_k \\ o = x^0 e_0 + \sum_{k=1}^7 x^k e_k & t+x_8 \end{array} \right) \in M_8^2 = SL_2(O)$

- Central cross encoding scalar $\phi$ and $Spin(9,1)$ spinor $\Psi = \left( \begin{array}{c} \psi^+ \\ \psi^- \end{array} \right)$, $J = \left( \begin{array}{cc} t-x_8 & \begin{array}{c} \psi^+ \\ \phi-2t \end{array} \\ \begin{array}{c} \psi^- \\ o \end{array} & \begin{array}{c} \bar{o} \\ t+x_8 \end{array} \end{array} \right) \in M_8^3 = SL_3(O)$

- Jordan product: $J_1 \cdot J_2 = \frac{1}{2} (J_1 J_2 + J_2 J_1)$ [1]

REF:

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**Freudenthal product:**

\[ J_1 \times J_2 = \frac{1}{2} (2J_1 \cdot J_2 - \text{Tr}(J_1)J_2 - \text{Tr}(J_2)J_1 + I(\text{Tr}(J_1)\text{Tr}(J_2) - \text{Tr}(J_1 \cdot J_2))) \]  

**Associator:**  

\[ [J_1, J_2, J_3] = (J_1 \cdot J_2) \cdot J_3 - J_1 \cdot (J_2 \cdot J_3) \]

**Left quasi multiplication:**  

\[ L_x : L_x(y) = x \cdot y \]

**Quadratic map:**  

\[ U_x = 2L_x^2 - L_x^2 \]

**Linearized map:**  

\[ V_{x,y} : V_{x,y}(z) = (U_{x+z} - U_x - U_z)(y) \]

**Trilinear map:**  

\[ \{x, y, z\} = V_{x,y}(z) = 2(L_{x,y} + [L_x, L_y])(z) \]

**Axioms:**

\[ A1 : U_x V_{y,x} = V_{x,y} U_x, \quad A2 : U_{U_{xy}} = U_x U_y U_x \]

**Jordan pair:**  

\[ x, y | A1 \& A2 \& V_{U_{xy},y} = V_{x,U_y x} \]

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**REF:**

Discrete Jordan Matrix

- Each octonion in $J$ can be encoded by its 9D coordinates in a 3x3 matrix.
- Induced by lattice coordinates they can be restricted to integer \([5]\)

\[
J' = \begin{pmatrix}
t - x_8 & \psi_+ & \bar{\phi} \\
\bar{\psi}_+ & \phi - 2t & \psi_-
\end{pmatrix}
\]

REF:

**F₄ action**

F₄ action is a derivation [6] on $\mathcal{M}_8^3$:

- An element of 52D algebra F₄ is represented by two traceless $H_+$ and $H_-$
- Its action [7] on $J=H + \Phi$ is $F₄(H_+, H_-)(J) = \delta J = [H_+, J, H_-]$
- Invariants are $I_1 = Tr(J)$, $I_2 = Tr(J^2)$, $I_3 = Det(J) = \frac{1}{3} Tr(J \cdot J \times J)$

**REF:**


**E₆ action**

E₆ action is a derivation [⁶] on $\mathcal{M}_8^3 \otimes \mathbb{C}$:

- An element of 78D algebra $E₆$ is represented by $H, H_+, H_- \in Tr_0(\mathcal{M}_8^3)$
- Its action [⁷] on $J$ is $E₆(H, H_+, H_-)(J) = \delta J = [H_+, J, H_-] + e_1 H_1 \cdot J$
- Invariants are $I_2 = Tr(J^2)$, $I_3 + \eta' I_3 = 3Det(J) = Tr(J \cdot (J \times J)^*)$, $I_4 = Tr((J \times J) \cdot (J^* \times J^*)^*)$

**E₆(−26) action**

An action on the reduced structure group is proposed in [⁸]

- $J = \Xi + \Psi + \Phi$
- $S = \frac{1}{8\pi} Tr \int d\sigma d\tau (\delta_\alpha \bar{\Xi} \delta^\alpha \Xi + \delta_\alpha \bar{\Psi} \delta^\alpha \Psi + \delta_\alpha \bar{\Phi} \delta^\alpha \Phi)$

REF:

**E₇ action**

An action of $E_7$ by a Freudenthal triple system on $E_8$ was proposed in [9]:

- 56D representation of $E_7$ as $\mathcal{M}^2_{27} = (\mathcal{M}^3_8)$

**E₈ action**

$E_8$ proposed action is a derivation on $\mathcal{M}^3_8 \otimes \mathcal{O}$:

- The action is extrapolated from Tits-Rosenfeld-Freudenthal magic square [10] expressed by Vinberg [10a] as:

$$L(A, J^3(B)) = \text{Der}(A) \bigoplus \text{Im}(A) \bigotimes \text{Tr}_0(J^3(B)) \bigoplus \text{Der}(J^3(B))$$ (11)

**REF:**


Magic star
From three $A_8$ lattices

Figure: Three $A_8$ lattices
Figure: Induced Fano plane
\[ E_8 = G_2 \times H_4? \]

A golden selective projection operates the \( H_4 \) folding

**Figure:** Rotate \( E_8 \) projection from \( G_2 \) to \( H_4 \) Coxeter plane
Quantum gravity
Quasi-lattice action

Figure: Elser-Sloane Quasicrystal triacontagonally projected
A the observer

Choose a tetrahedron, select a vertex in it, select an operation

► Operation $F_4$ involves two $E_8$ vertices and updates one $E_8$ vertex
► Operation $E_6$ involves three vertices and updates two
► Operation $E_7$ needs choosing a top in the tetrahedron, involves seven vertices
► Operation $E_8$ involves the full magic star

B the observed

All Jordan matrices affected to lattice vertices are initially blank

P the observation

► Once the observer and its operation chosen, selected vertices, if blank, are initialized
► The operation is performed and vertices are updated
Figure: Elser-Sloane Quasicrystal with numbered 600-cells
Figure: Elser-Sloane Quasicrystal unflattened
Quantum gravity
Quasi-lattice action

Figure: 30-ring
Quantum gravity
Quasi-lattice action

**Figure**: Two rings
Quantum gravity
Quasi-lattice action

Figure: Two rings
Quantum gravity
Quasi-lattice action

Figure: Two rings
Quantum gravity
Quasi-lattice action

Figure: Two rings
Quantum gravity
Quasi-lattice action

Figure: Two rings
Quantum gravity
Particle Model

Figure: Gosset polytope projected to Tony’s new model
Quantum gravity
Particle Model

Figure: $\varepsilon_8$ contracted as $\mathfrak{h}_{92} \rtimes a_7$. 
Thank you!
Quantum gravity
Particle Model

Figure: $e_8$ contracted as $h_{92} \times a_7$, superposed to a Kongokai "Diamond" mandala (Tō-ji, Kyoto, 9th century - Credit https://commons.wikimedia.org/wiki/File:Kongokai.jpg). “It is like a diamond with tens of thousands of facets,” Bertram Kostant, an emeritus professor of math at M.I.T., said. “It is easy to arrive at the feeling that a final understanding of the universe must somehow involve E8, or, otherwise put, nature would be foolish not to utilize E8.” https://www.newyorker.com/magazine/2008/07/21/surfing-the-universe